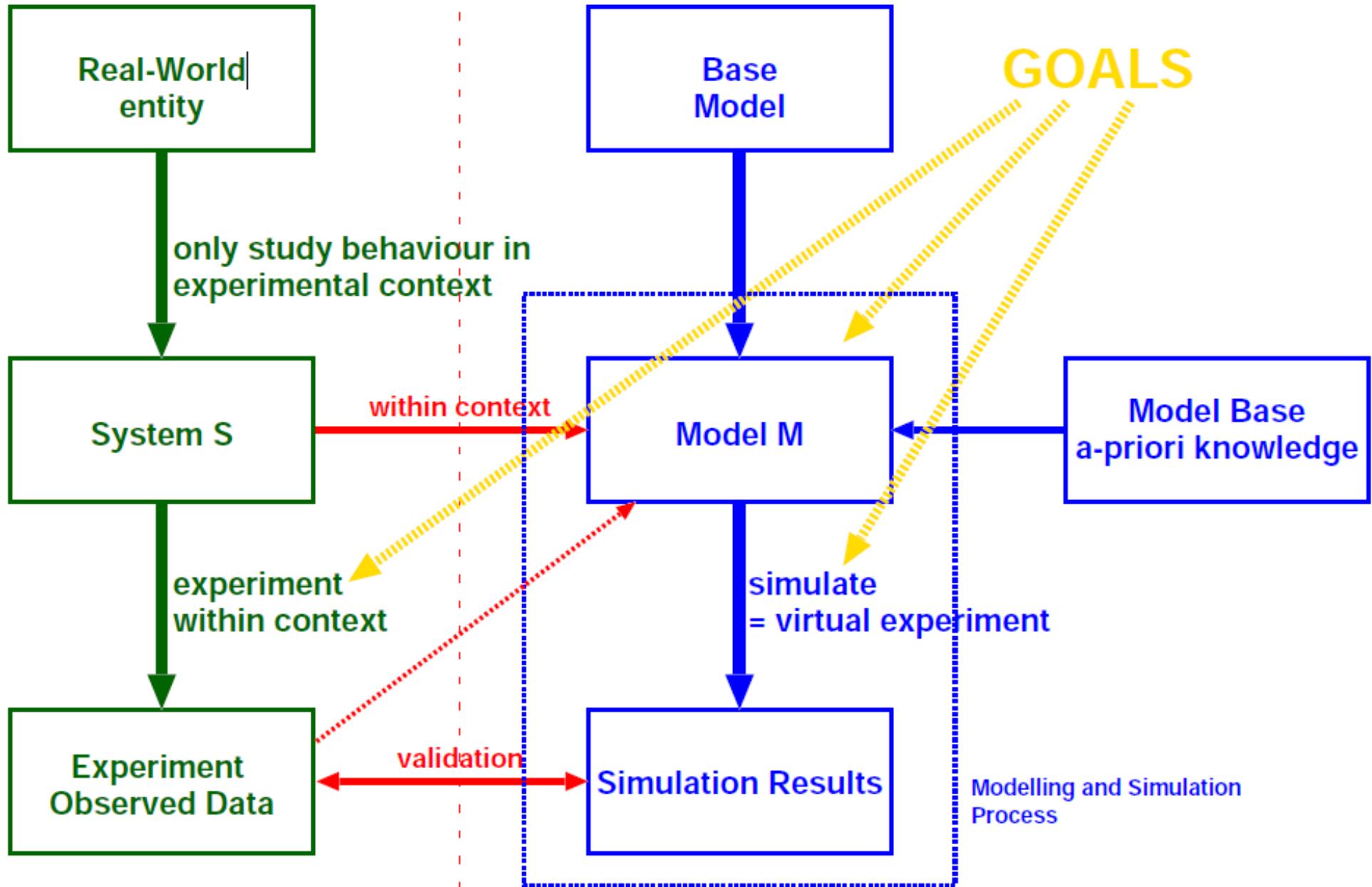


# Modelling of Physical Systems (for Computer Scientists)

Hans Vangheluwe

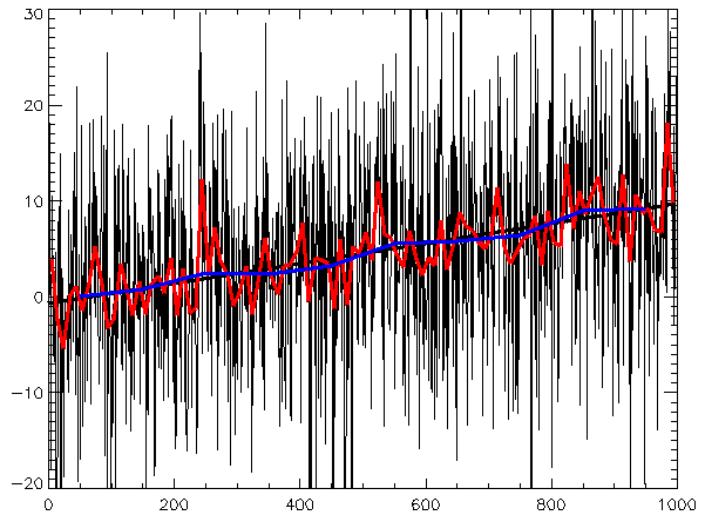
# REALITY

# MODEL

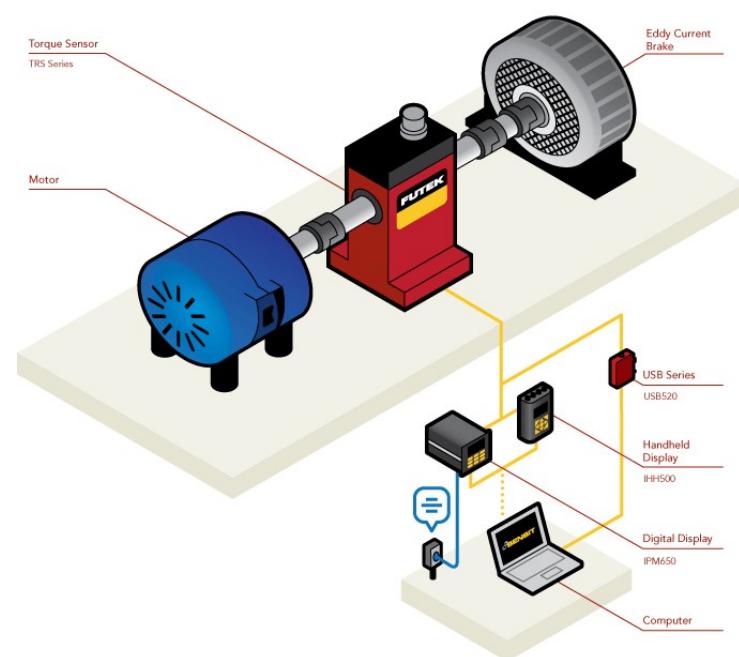
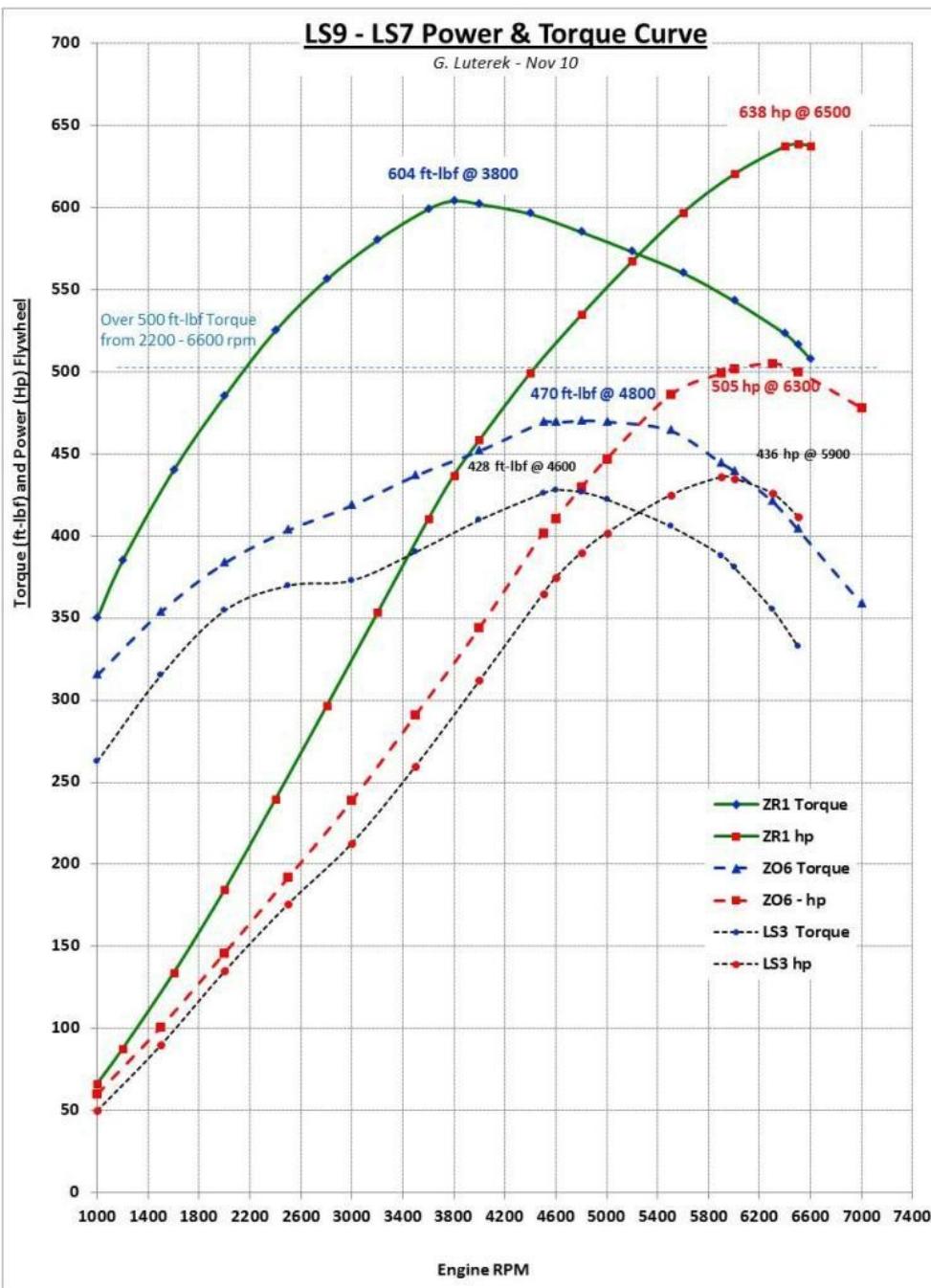


# models based on measurements

- instance (technology) – specific
- high (experimentation) cost
- may not even be possible to measure
- allows reproducing data, no extrapolation;  
no insight/explanation
- inductive vs. deductive modelling workflow  
science vs. engineering, usually combination



# Torque Curve “model” (measured)



# mathematical model + data



[www.vishay.com](http://www.vishay.com)

CH

Vishay Sfernice

## High Frequency 50 GHz Thin Film Chip Resistor



CH02016  
(flip chip)



CH0402  
(flip chip)



CH0603  
(flip chip)

### FEATURES

- Operating frequency 50 GHz
- Thin film microwave resistors
- Flip chip, wraparound or one face termination
- Small size, down to 20 mils by 16 mils
- Edged trimmed block resistors
- Pure alumina substrate (99.5 %)
- Ohmic range: 10R to 500R
- Design kits available
- Small internal reactance (LC down to  $1 \times 10^{-24}$ )
- Tolerance 1 %, 2 %, 5 %
- TCR: 100 ppm/ $^{\circ}\text{C}$  in (-55  $^{\circ}\text{C}$ , +155  $^{\circ}\text{C}$ ) temperature range
- TCR: 50 ppm/ $^{\circ}\text{C}$  available upon request for 10  $\Omega$  to 150  $\Omega$  ohmic range
- Material categorization: for definitions of compliance please see [www.vishay.com/doc?99912](http://www.vishay.com/doc?99912)



RoHS  
COMPLIANT  
HALOGEN  
**FREE**  
**GREEN**  
(5-2008)

### ADDITIONAL RESOURCES



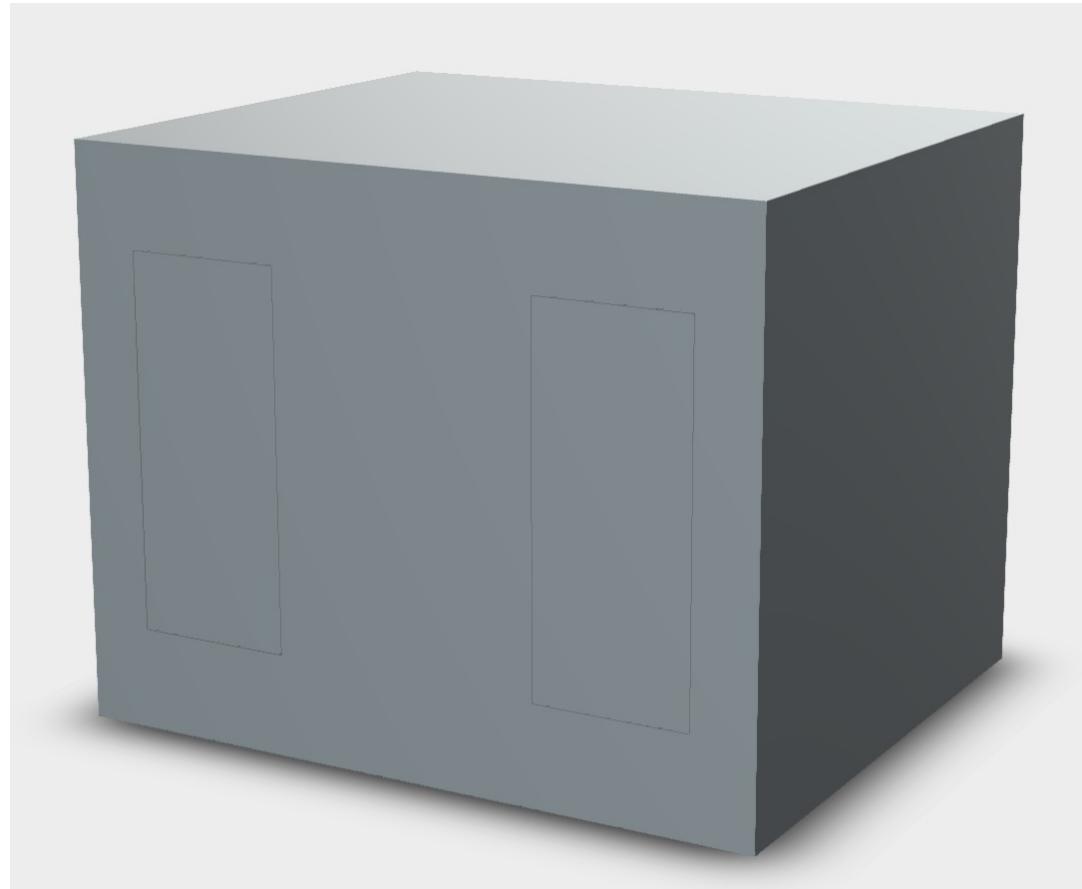
Those miniaturized components are designed in such a way that their internal reactance is very small. When correctly mounted and utilized, they function as almost pure resistors on a very large range of frequency, up to 50 GHz.

### STANDARD ELECTRICAL SPECIFICATIONS

| MODEL   | SIZE  | RESISTANCE RANGE $\Omega$ | RATED POWER $P_n$ W | LIMITING ELEMENT VOLTAGE V | TOLERANCE $\pm$ % | TEMPERATURE COEFFICIENT $\pm$ ppm/ $^{\circ}\text{C}$ |
|---------|-------|---------------------------|---------------------|----------------------------|-------------------|---|
| CH02016 | 02016 | 10 to 500                 | 0.030               | 30                         | 2, 5              | 100 (50 upon request)                                 |
| CH0402  | 0402  | 10 to 500                 | 0.050               | 37                         | 1, 2, 5           | 100 (50 upon request)                                 |
| CH0603  | 0603  | 10 to 500                 | 0.125               | 50                         | 1, 2, 5           | 100 (50 upon request)                                 |

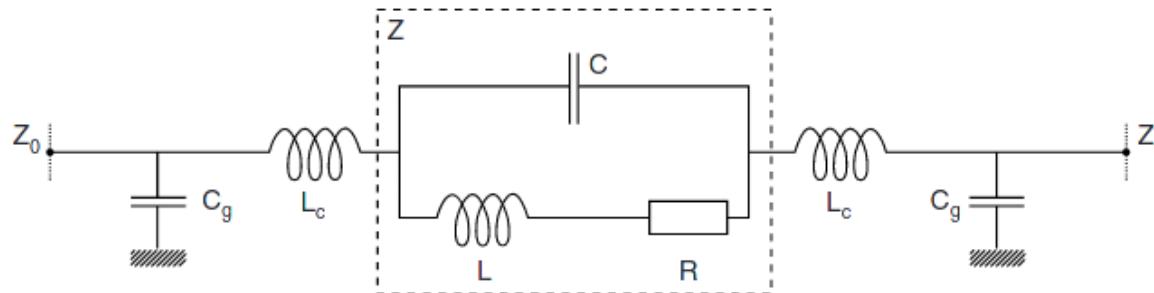
# ISO 10303-21 STEP 3D CAD file

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ISO-10303-21;
HEADER;
FILE_DESCRIPTION((
'CAX-IF Rec.Pracs.--Model Styling and Organization--1.5---2016-08-15',
'CAX-IF Rec.Pracs.--Geometric and Assembly Validation
Properties---4.4---2016-08-17',
'CAX-IF Rec.Pracs.--User Defined Attributes---1.5---2016-08-15',
'CAX-IF Rec.Pracs.--External References---2.1---2005-01-19'),'2;1'
);
FILE_NAME('CH02016_P.stp','2017-11-14T12:13:16','(Unspecified)',(
'Unspecified'),'CAD Exchanger 3.3.1 (www.cadexchanger.com)',
'CAD Exchanger 3.3.1','');
FILE_SCHEMA({'AUTOMOTIVE DESIGN { 1 0 10303 214 1 1 1 }'});
ENDSEC;
DATA;
#1 = APPLICATION_PROTOCOL_DEFINITION('international standard',
'automotive_design',2000,#2);
#2 = APPLICATION_CONTEXT(
'core data for automotive mechanical design processes');
#3 = SHAPE_DEFINITION_REPRESENTATION(#4,#10);
#4 = PRODUCT_DEFINITION_SHAPE('',$,#5);
#5 = PRODUCT_DEFINITION('design','','#6,#9);
#6 = PRODUCT_DEFINITION_FORMATION('','','#7);
#7 = PRODUCT('67','67','','#8);
#8 = PRODUCT_CONTEXT('#2,'mechanical');
#9 = PRODUCT_DEFINITION_CONTEXT('part definition',#2,'design');
#10 = ADVANCED_BREP_SHAPE REPRESENTATION('',(##16,#154),#11);
#11 = ( GEOMETRIC REPRESENTATION_CONTEXT(3)
GLOBAL_UNCERTAINTY_ASSIGNED_CONTEXT((#12)) GLOBAL_UNIT_ASSIGNED_CONTEXT(
(#13,#14,#15)) REPRESENTATION_CONTEXT('','') );
#12 = UNCERTAINTY_MEASURE_WITH_UNIT(LENGTH_MEASURE(1.E-007),#13,'',
'maximum tolerance');
#13 = ( LENGTH_UNIT() NAMED_UNIT(*) SI_UNIT(.MILLI.,.METRE.) );
#14 = ( NAMED_UNIT(*) PLANE_ANGLE_UNIT() SI_UNIT($,.RADIAN.) );
#15 = ( NAMED_UNIT(*) SI_UNIT($,.STERADIAN.) SOLID_ANGLE_UNIT() );
#16 = MANIFOLD_SOLID_BREP('67',#17);
#17 = CLOSED_SHELL('',(##18,#54,#90,#110,#126,#142));
#18 = ADVANCED_FACE('',(##24),#19,.T.);
#19 = B_SPLINE_SURFACE_WITH_KNOTS('',1,1,(#20,#21)
,(#22,#23
)),.UNSPECIFIED.,.F.,.F.,.U.,(2,2),(2,2),(-0.251,0.251),(-0.201,
0.201),.PIECEWISE_BEZIER_KNOTS. );
#20 = CARTESIAN_POINT('',(-1.E-003,-1.E-003,0.42));
#21 = CARTESIAN_POINT('',(-1.E-003,0.401,0.42));
#22 = CARTESIAN_POINT('',(0.501,-1.E-003,0.42));
#23 = CARTESIAN_POINT('',(0.501,0.401,0.42));
#24 = FACE_BOUND('',#25,.T. );
#25 = EDGE_LOOP('',(##26,#35,#42,#49));
#26 = ORIENTED_EDGE('','*',*,#27,.T. );
#27 = EDGE_CURVE('',(##31,#33,#28,.T. );
#28 = B_SPLINE_CURVE_WITH_KNOTS('',1,(##29,#30),.UNSPECIFIED.,.F.,.U.,(2,
2),(-0.2,0.2),.PIECEWISE_BEZIER_KNOTS. );
#29 = CARTESIAN_POINT('',(0.5,0.E+000,0.42));
#30 = CARTESIAN_POINT('',(0.5,0.4,0.42));
#31 = VERTEX_POINT('',#32);
#32 = CARTESIAN_POINT('',(0.5,0.E+000,0.42));
#33 = VERTEX_POINT('',#34);
#34 = CARTESIAN_POINT('',(0.5,0.4,0.42));
#35 = ORIENTED_EDGE('','*',*,#36,.T. );
#36 = EDGE_CURVE('',(##33,#40,#37,.T. );
#37 = B_SPLINE_CURVE_WITH_KNOTS('',1,(##38,#39),.UNSPECIFIED.,.F.,.U.,(2,
2),(-0.25,0.25),.PIECEWISE_BEZIER_KNOTS. );
#38 = CARTESIAN_POINT('',(0.5,0.4,0.42));
#39 = CARTESIAN_POINT('',(0.E+000,0.4,0.42));
#40 = VERTEX_POINT('',#41);
#41 = CARTESIAN_POINT('',(0.E+000,0.4,0.42));
#42 = ORIENTED_EDGE('','*',*,#43,.T. );
#43 = EDGE_CURVE('',(##40,#47,#44,.T. );
#44 = B_SPLINE_CURVE_WITH_KNOTS('',1,(##45,#46),.UNSPECIFIED.,.F.,.U.,(2,
2),(-0.2,0.2),.PIECEWISE_BEZIER_KNOTS. );
#45 = CARTESIAN_POINT('',(0.E+000,0.4,0.42));
#46 = CARTESIAN_POINT('',(0.E+000,0.E+000,0.42));
#47 = VERTEX_POINT('',#48);
...
)
```



AUTODESK® VIEWER > CH02016\_P.stp

## TYPICAL HIGH FREQUENCY PERFORMANCE ELECTRICAL MODEL



|       |  |
|-------|--|
| C     | Internal shunt capacitance             |
| L     | Internal inductance                    |
| R     | Resistance                             |
| Z     | Internal impedance ( $R$ , $L$ , $C$ ) |
| $L_c$ | External connection inductance         |
| $C_g$ | External capacitance to ground         |

### INTERNAL IMPEDANCE CURVES

The complex impedance of the chip resistor is given by the following equations:

$$Z = \frac{R + j\omega(L - R^2C - L^2C\omega^2)}{1 + C[(R^2C - 2L)\omega^2 + L^2C\omega^4]}$$

$$\frac{[Z]}{R} = \frac{1}{1 + C[(R^2C - 2L)\omega^2 + L^2C\omega^4]} \times \sqrt{1 + \left[ \frac{\omega(L - R^2C - L^2C\omega^2)}{R} \right]^2}$$

$$\theta = \tan^{-1} \frac{\omega(L - R^2C - L^2C\omega^2)}{R}$$

#### Notes

- $\omega = 2 \times \pi \times f$
- $f$ : frequency

R, L and C are relevant to the chip resistor itself.

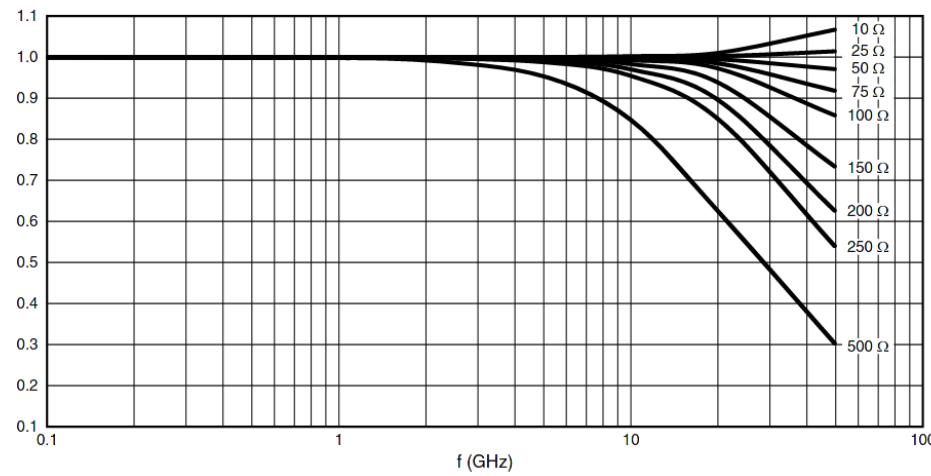
$L_c$  and  $C_g$  also depend on the way the chip resistor is mounted.

It is important to notice that after assembly the external reactance of  $L_c$  and  $C_g$  will be combined to internal reactance of L and C. This combination can upgrade or downgrade the HF behavior of the component.

This is why we are displaying three sets of data:

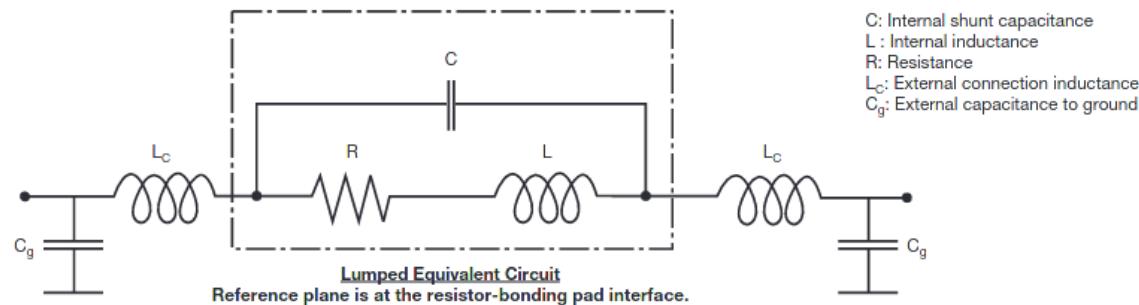
- $\frac{[Z]}{R}$  versus frequency curves which aim to show at a glance the intrinsic HF performance of a given chip resistor

- $\frac{[Z_{total}]}{R}$  versus frequency curves which aim to show the behavior of the chip resistor when mounted

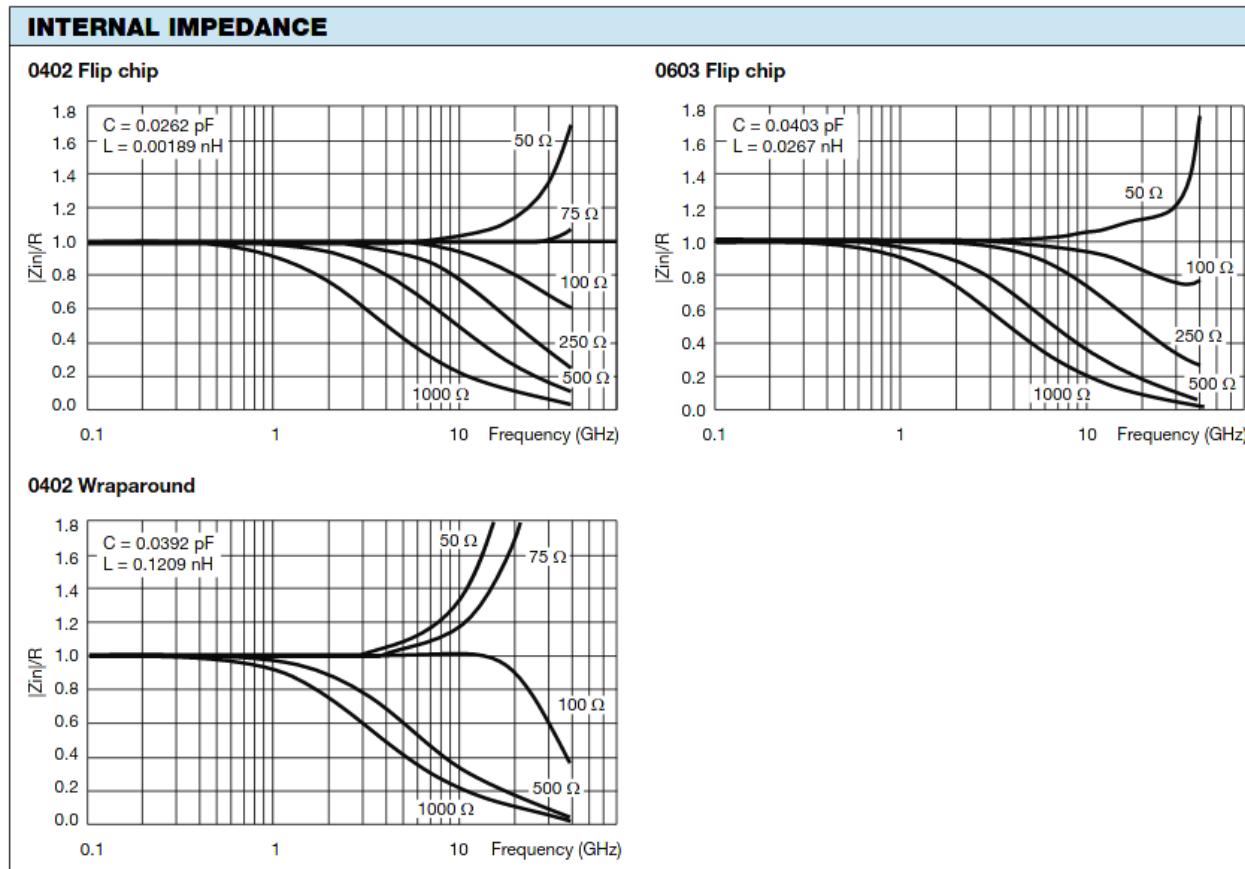


Internal impedance curve for 02016 size (F and P terminations)

## TYPICAL HIGH FREQUENCY PERFORMANCE ELECTRICAL MODEL AND TESTING



The lumped circuit above was used to model the data at the bonding pad-resistor reference plane. High frequency testing was performed by Modelithics, Inc. on parts mounted to quartz test boards. Quartz test boards were chosen to minimize the contribution of the board effects at high frequencies. Future testing will be performed on various industry standard board types. Vishay in partnership with Modelithics, Inc. will develop substrate scalable models for the FC series resistors. These models will be available for industry standard design software packages and will allow the designer to accurately model their wireless and microwave printed boards.



# models based on Laws of Physics

$$\begin{aligned}
 j &= \sum_{i=1}^3 \frac{\partial L}{\partial \dot{x}_i} Q[x_i] - f \\
 &= m \sum_i \dot{x}_i^2 - \left[ \frac{m}{2} \sum_i \dot{x}_i^2 - V(x) \right] \\
 &= \frac{m}{2} \sum_i \dot{x}_i^2 + V(x).
 \end{aligned}$$

$$\sum_i \left( \frac{d}{dt} \left( \frac{\partial \mathcal{L}_\alpha}{\partial \dot{q}_i} \frac{\partial q_i}{\partial \alpha} \right) \right)$$


$$\begin{aligned}
 \frac{\partial \alpha}{\partial t} &= \sum_i \left( \frac{\partial \mathcal{L}_\alpha}{\partial q_i} \frac{\partial q_i}{\partial \alpha} + \frac{\partial \mathcal{L}_\alpha}{\partial \dot{q}_i} \frac{\partial \dot{q}_i}{\partial \alpha} \right) \\
 0 &= \sum_i \left( \frac{d}{dt} \left( \frac{\partial \mathcal{L}_\alpha}{\partial \dot{q}_i} \frac{\partial q_i}{\partial \alpha} \right) + \frac{\partial \mathcal{L}_\alpha}{\partial q_i} \frac{\partial \dot{q}_i}{\partial \alpha} \right)
 \end{aligned}$$

Invariante Variationsprobleme.

(F. Klein zum fünfzigjährigen Doktorjubiläum.)

Von

**Emmy Noether** in Göttingen.

Vorgelegt von F. Klein in der Sitzung vom 26. Juli 1918<sup>1)</sup>.

Es handelt sich um Variationsprobleme, die eine kontinuierliche Gruppe (im Lieschen Sinne) gestatten; die daraus sich ergebenden Folgerungen für die zugehörigen Differentialgleichungen finden ihren allgemeinsten Ausdruck in den in § 1 formulierten, in den folgenden Paragraphen bewiesenen Sätzen. Über diese aus Variationsproblemen entspringenden Differentialgleichungen lassen sich viel präzisere Aussagen machen als über beliebige, eine Gruppe gestattende Differentialgleichungen, die den Gegenstand der Lieschen Untersuchungen bilden. Das folgende beruht also auf einer Verbindung der Methoden der formalen Variationsrechnung mit denen der Lieschen Gruppentheorie. Für spezielle Gruppen und Variationsprobleme ist diese Verbindung der Methoden nicht neu; ich erwähne Hamel und Herglotz für spezielle endliche, Lorentz und seine Schüler (z. B. Fokker), Weyl und Klein für spezielle unendliche Gruppen<sup>2)</sup>. Insbesondere sind die zweite Kleinsche Note und die vorliegenden Ausführungen gegenseitig durch einander beein-

1) Die endgültige Fassung des Manuskriptes wurde erst Ende September eingereicht.

2) Hamel: Math. Ann. Bd. 59 und Zeitschrift f. Math. u. Phys. Bd. 50. Herglotz: Ann. d. Phys. (4) Bd. 36, bes. § 9, S. 511. Fokker, Verslag d. Amsterdamer Akad., 27./1. 1917. Für die weitere Literatur vergl. die zweite Note von Klein: Göttinger Nachrichten 19. Juli 1918.

In einer eben erschienenen Arbeit von Kneser (Math. Zeitschrift Bd. 2) handelt es sich um Aufstellung von Invarianten nach ähnlicher Methode.

Kgl. Ges. d. Wiss. Nachrichten. Math.-phys. Klasse, 1918. Heft 2.

**Noether's theorem** or **Noether's first theorem** states that every **differentiable symmetry** of the action of a physical system has a corresponding **conservation law**.<sup>[1]</sup>

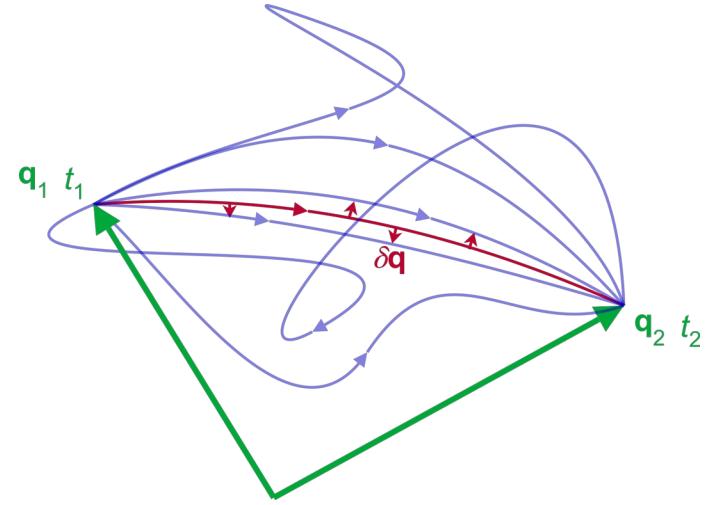
The theorem was proven by mathematician **Emmy Noether** in 1915 and published in 1918,<sup>[2]</sup> after a special case was proven by **E. Cosserat** and **F. Cosserat** in 1909.<sup>[3]</sup>

The action of a physical system is the **integral over time** of a **Lagrangian** function (which may be an **integral over space** of a **Lagrangian density function**), from which the system's behavior can be determined by the **principle of least action**. This theorem only applies to continuous and smooth symmetries over **physical space**.

Application of Noether's theorem allows physicists to gain powerful insights into any general theory in physics, by just analyzing the various transformations that would make the form of the laws involved invariant. For example:

- the invariance of physical systems with respect to spatial **translation** (in other words, that the laws of physics do not vary with locations in space) gives the law of conservation of **linear momentum**;
- invariance with respect to **rotation** gives the law of conservation of **angular momentum**;
- invariance with respect to **time translation** gives the well-known **law of conservation of energy**

In **quantum field theory**, the analog to Noether's theorem, the **Ward-Takahashi identity**, yields further conservation laws, such as the conservation of **electric charge** from the invariance with respect to a change in the **phase factor** of the **complex field** of the charged particle and the associated **gauge** of the **electric potential** and **vector potential**.



In non-relativistic physics, the **principle of least action** – or, more accurately, the **principle of stationary action** – is a [variational principle](#) that, when applied to the [action](#) of a [mechanical system](#), can be used to obtain the [equations of motion](#) for that system by stating a system follows the path where the average difference between the kinetic energy and potential energy is minimized or maximized over any time period. It is called stable if minimized. In relativity, a different average must be minimized or maximized. The principle can be used to derive [Newtonian](#), [Lagrangian](#), and [Hamiltonian equations of motion](#).

The starting point is the *action*, denoted  $\mathcal{S}$  (calligraphic S), of a physical system. It is defined as the integral of the *Lagrangian*  $L$  between two instants of *time*  $t_1$  and  $t_2$  - technically a *functional* of the  $N$  *generalized coordinates*  $\mathbf{q} = (q_1, q_2 \dots q_N)$  which define the *configuration* of the system:

$$\mathcal{S}[\mathbf{q}(t)] = \int_{t_1}^{t_2} L(\mathbf{q}(t), \dot{\mathbf{q}}(t), t) dt$$

where the dot denotes the *time derivative*, and  $t$  is time.

Mathematically the principle is<sup>[11][12][13]</sup>

$$\delta\mathcal{S} = 0$$

where  $\delta$  (Greek lowercase *delta*) means a *small* change. In words this reads:<sup>[10]</sup>

*The path taken by the system between times  $t_1$  and  $t_2$  is the one for which the action is stationary (no change) to first order.*

### \* **Conservation of Mass-Energy:**

The total energy in a closed or isolated system is constant, no matter what happens.

### \* **Conservation of Momentum:**

The total momentum in a closed or isolated system remains constant. An alternative of this is the law of conservation of angular momentum.

### \* **Newton's Law of Gravity:**

Explains the attractive force between a pair of masses. In the twentieth century, it became clear that this is not the whole story, as **Einstein's theory of general relativity** has provided a more comprehensive explanation for the phenomenon of gravity.

### \* **Newton's Three Laws of Motion:**

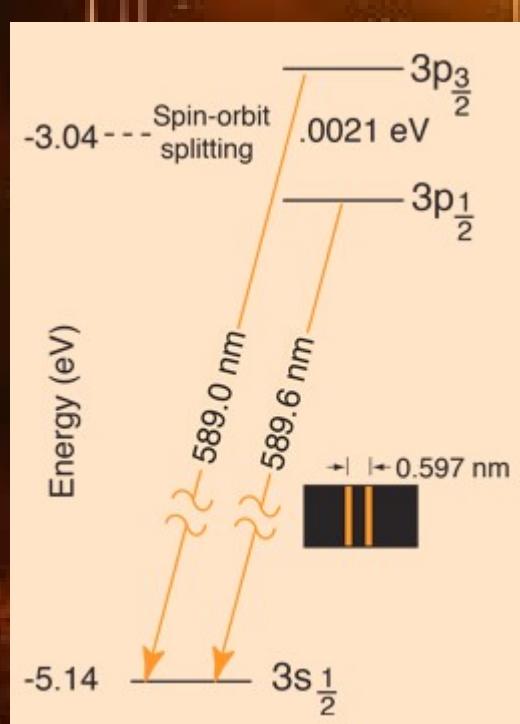
Fundamental relationship between the acceleration of an object and the total forces acting upon it.

- First Law states that in order for the motion of an object to change, a force must act upon it, a concept generally called inertia.
- Second Law defines the relationship between acceleration, force, and mass.  $F = m a$  ?
- Third Law states that any time a force acts from one object to another, there is an equal force acting back on the original object.

\* ...



GPS: relativistic



Sodium lamp: quantum



# “distributed parameter” models (based on Laws of Physics)

Model Builder

busbar\_geom.mph (root)  
Global Definitions  
Parameters 1  
Common Model Inputs  
Materials  
Component 1 (comp1)  
Definitions  
Geometry 1  
Materials  
Electric Currents (ec)  
Heat Transfer in Solids (ht)  
Multiphysics  
Mesh 1  
Study 1  
Results

Properties

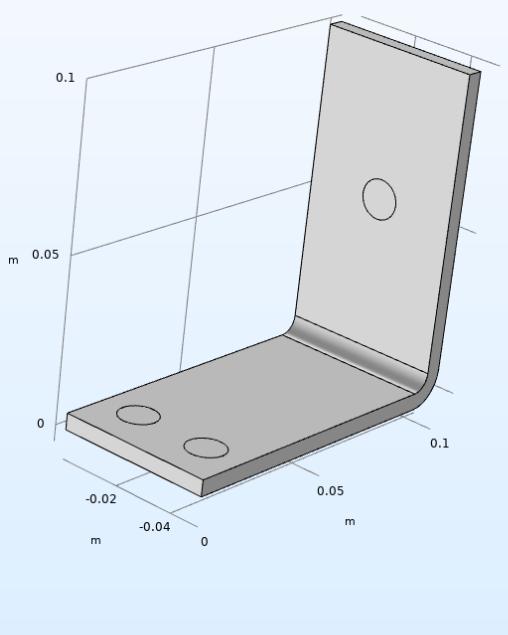
Label: Parameters 1

Parameters

| Name  | Expression | Value                   | Description               |
|-------|------------|-------------------------|---------------------------|
| L     | 9[cm]      | 0.09 m                  | Length                    |
| rad_1 | 6[mm]      | 0.006 m                 | Bolt radius               |
| tbb   | 5[mm]      | 0.005 m                 | Thickness                 |
| wbb   | 5[cm]      | 0.05 m                  | Width                     |
| mh    | 3[mm]      | 0.003 m                 | Maximum element size      |
| htc   | 5[W/m^2·K] | 5 W/(m <sup>2</sup> ·K) | Heat transfer coefficient |
| Vtot  | 20[mV]     | 0.02 V                  | Applied voltage           |

Name:  
Expression:  
Description:  
Visibility

Graphics



The image shows a 3D CAD model of a busbar component. The model is a rectangular plate with a U-shaped cutout at one end. The overall length is 0.09 meters (L). The thickness of the plate is 0.005 meters (tbb). There are three circular holes along the vertical edge of the U-cutout. The width of the plate is 0.05 meters (wbb). A coordinate system (x, y, z) is shown at the bottom left corner of the model. The model is displayed in a 3D view with axes labeled from -0.04 to 0.1 meters.

COMSOL MULTIPHYSICS®

parametrized model

**Model Builder**

- busbar\_geom.mph (root)
  - Global Definitions
  - Parameters 1
  - Common Model Inputs
  - Materials
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
      - Work Plane 1 (wp1)
        - Plane Geometry
          - Rectangle 1 (r1)
          - Rectangle 2 (r2)
          - Difference 1 (dif1)
          - Fillet 1 (fil1)
          - Fillet 2 (fil2)
        - View 2
        - Extrude 1 (ext1)
      - Work Plane 2 (wp2)
        - Plane Geometry
          - Circle 1 (c1)
        - View 3
        - Extrude 2 (ext2)
      - Work Plane 3 (wp3)
        - Plane Geometry
          - Circle 1 (c1)
          - Copy 1 (copy1)
        - View 4
        - Extrude 3 (ext3)
        - Form Union (fin)
    - Materials
    - Electric Currents (ec)
    - Heat Transfer in Solids (ht)
    - Multiphysics
      - Mesh 1
  - Study 1
  - Results

**Properties**

Geometry  
Build All

Label: Geometry 1

Units

Scale values when changing units

Length unit: m

Angular unit: Degrees

Advanced

Geometry representation: COMSOL kernel

Default repair tolerance: Automatic

Automatic rebuild

**Graphics**

# geometry

Model Builder

busbar\_geom.mph (root)

- Global Definitions
- Component 1 (comp1)
  - Definitions
  - Geometry 1
  - Materials
    - Copper (mat1)
    - Titanium beta-21S (mat2)
  - Electric Currents (ec)
  - Heat Transfer in Solids (ht)
  - Multiphysics
  - Mesh 1
- Study 1
- Results

Settings Properties

Material

Label: Copper

Geometric Entity Selection

Geometric entity level: Domain

Selection: All domains

Active

|    |                |
|----|----------------|
| ON | 1              |
|    | 2 (overridden) |
|    | 3 (overridden) |
|    | 4 (overridden) |
|    | 5 (overridden) |
|    | 6 (overridden) |

Override

Material Properties

Material Contents

| Property   | Variable         | Value                    | Unit              | Property group                   |
|--|------------------|--------------------------|-------------------|----------------------------------|
| <input checked="" type="checkbox"/> Electrical conductivity            | $\sigma_{iso}$   | 5.999e7                  | S/m               | Basic                            |
| <input checked="" type="checkbox"/> Heat capacity at constant pressure | $C_p$            | 385J/(kg*K)              | J/(kg·K)          | Basic                            |
| <input checked="" type="checkbox"/> Relative permittivity              | $\epsilon_{iso}$ | 1                        | 1                 | Basic                            |
| <input checked="" type="checkbox"/> Density                            | $\rho$           | 8960[kg/m <sup>3</sup> ] | kg/m <sup>3</sup> | Basic                            |
| <input checked="" type="checkbox"/> Thermal conductivity               | $k_{iso}$        | 400[W/(m·K)]             | W/(m·K)           | Basic                            |
| Relative permeability  | $\mu_{iso}$      | 1                        | 1                 | Basic                            |
| Coefficient of thermal expansion                                       | $\alpha_{iso}$   | 17e-6[1/K]               | 1/K               | Basic                            |
| Young's modulus  | E                | 110e9[Pa]                | Pa                | Young's modulus and Poisson's... |
| Poisson's ratio  | $\nu$            | 0.35                     | 1                 | Young's modulus and Poisson's... |
| Reference resistivity  | $\rho_0$         | 1.72e-8[ohm·m]           | Q·m               | Linearized resistivity           |
| Resistivity temperature coefficient                                    | $\alpha$         | 0.0039(1/K)              | 1/K               | Linearized resistivity           |
| Reference temperature  | Tref             | 298[K]                   | K                 | Linearized resistivity           |

Appearance

Graphics

# material properties

Model Builder

- busbar\_geom.mph (root)
  - Global Definitions
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
    - Materials
      - Copper (mat1)
      - Titanium beta-21S (mat2)
    - Electric Currents (ec)
    - Heat Transfer in Solids (ht)
    - Multiphysics
    - Mesh 1
  - Study 1
  - Results

Settings Properties

Material

Label: Titanium beta-21S

Geometric Entity Selection

Geometric entity level: Domain

Selection: Manual

|        |   |
|--------|---|
| ON     | 2 |
|        | 3 |
| Active | 4 |
|        | 5 |
|        | 6 |
|        | 7 |

Override

Material Properties

Material Contents

| Property                           | Variable         | Value   | Unit              | Property group                      |
|------------------------------------|------------------|---------|-------------------|-------------------------------------|
| Electrical conductivity            | $\sigma_{iso}$   | 7.407e5 | S/m               | Basic                               |
| Heat capacity at constant pressure | $C_p$            | 710     | J/(kg·K)          | Basic                               |
| Relative permittivity              | $\epsilon_{iso}$ | 1       | 1                 | Basic                               |
| Density                            | $\rho$           | 4940    | kg/m <sup>3</sup> | Basic                               |
| Thermal conductivity               | $k_{iso}$        | 7.5     | W/(m·K)           | Basic                               |
| Relative permeability              | $\mu_{iso}$      | 1       | 1                 | Basic                               |
| Coefficient of thermal expansion   | $\alpha_{iso}$   | 7.06e-6 | 1/K               | Basic                               |
| Young's modulus                    | E                | 105e9   | Pa                | Young's modulus and Poisson's ratio |
| Poisson's ratio                    | $\nu$            | 0.33    | 1                 | Young's modulus and Poisson's ratio |

Appearance

Graphics

# material properties

**Model Builder**

- busbar\_geom.mph (root)
  - Global Definitions
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
    - Materials
  - Electric Currents (ec)
    - Current Conservation 1
    - Electric Insulation 1
    - Initial Values 1
    - Electric Potential 1
    - Ground 1
  - Heat Transfer in Solids (ht)
  - Multiphysics
  - Mesh 1
- Study 1
- Results

**Settings Properties**

**Electric Currents**

Label: Electric Currents  
Name: ec

Domain Selection

Selection: All domains

|                                     |   |
|-------------------------------------|---|
| <input checked="" type="checkbox"/> | 1 |
|                                     | 2 |
|                                     | 3 |
|                                     | 4 |
|                                     | 5 |
|                                     | 6 |

Active

Equation

Equation form:  
Study controlled

Show equation assuming:  
Study 1, Time Dependent

$$\nabla \cdot \mathbf{J} = Q_{j,v}$$

$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_e$$

$$\mathbf{E} = -\nabla V$$

Manual Terminal Sweep Settings

Reference impedance:  
 $Z_{ref}$  | 50[ohm]

Activate manual terminal sweep

Physics-Controlled Mesh

Enable

Discretization

Dependent Variables

**Graphics**

# Laws of Physics

Model Builder

busbar\_geom.mph (root)

- Global Definitions
- Component 1 (comp 1)
  - Definitions
  - Geometry 1
  - Materials
  - Electric Currents (ec)
    - Current Conservation 1
    - Electric Insulation 1
    - Initial Values 1
    - Electric Potential 1
    - Ground 1
  - Heat Transfer in Solids (ht)
  - Multiphysics
  - Mesh 1
- Study 1
- Results

Settings Properties

Current Conservation

Label: Current Conservation 1

Domain Selection

Selection: All domains

Active 3  
1  
2  
4  
5  
6

Override and Contribution

Equation

Show equation assuming:  
Study 1, Time Dependent

$$\nabla \cdot \mathbf{J} = Q_{j,v}$$

$$\mathbf{J} = \sigma \mathbf{E} + \mathbf{J}_e$$

$$\mathbf{E} = -\nabla V$$

Model Inputs

Temperature:  
T Temperature (emh1)

Material Type

Material type:  
Nonsolid

Coordinate System Selection

Coordinate system:  
Global coordinate system

Conduction Current

Electrical conductivity:  
 $\sigma$  From material

Electric Field

Constitutive relation:  
Relative permittivity

$$\mathbf{D} = \epsilon_0 \epsilon_r \mathbf{E}$$

Relative permittivity:  
 $\epsilon_r$  From material

Graphics

# Laws of Physics

**Model Builder**

- busbar.geom.mph (root)
  - Global Definitions
  - Component 1 (comp 1)
    - Definitions
    - Geometry 1
    - Materials
    - Electric Currents (ec)
      - Current Conservation 1
      - Electric Insulation 1
      - Initial Values 1
      - Electric Potential 1
      - Ground 1
    - Heat Transfer in Solids (ht)
    - Multiphysics
    - Mesh 1
  - Study 1
  - Results

**Properties**

**Electric Insulation**

Label: Electric Insulation 1

Boundary Selection

Selection: All boundaries

Active

|    |   |
|----|---|
| ON | 1 |
|    | 2 |
|    | 3 |
|    | 4 |
|    | 5 |
|    | 6 |

Override and Contribution

Overridden by:

- Ground 1
- Electric Potential 1

Overrides:

Contributes with:

Equation

Show equation assuming:

Study 1, Time Dependent

$\mathbf{n} \cdot \mathbf{J} = 0$

**Graphics**

# Laws of Physics

Model Builder

busbar\_geom.mph (root)

- Global Definitions
- Component 1 (comp 1)
  - Definitions
  - Geometry 1
  - Materials
- Electric Currents (ec)
  - Current Conservation 1
  - Electric Insulation 1
  - Initial Values 1
  - Electric Potential 1
  - Ground 1
- Heat Transfer in Solids (ht)
- Multiphysics
- Mesh 1

Study 1

Results

Properties

Initial Values

Label: Initial Values 1

Domain Selection

Selection: All domains

Active

Override and Contribution

Initial Values

Electric potential: V 0

Graphics

The graphics window displays a 3D model of a busbar component. The model is shown in a coordinate system with axes x, y, and z. Dimensions are indicated as follows: height 0.1 m, width 0.05 m, thickness 0.05 m. A circular hole with a diameter of 0.02 m is located on the top surface. Two small circular features are at the base. The model is shown in a wireframe style with a blue shaded top surface.

# initial values

Model Builder

busbar\_geom.mph (root)

- Component 1 (comp1)
  - Definitions
  - Geometry 1
  - Materials
  - Electric Currents (ec)
    - Current Conservation 1
    - Electric Insulation 1
    - Initial Values 1
    - Electric Potential 1
    - Ground 1
  - Heat Transfer in Solids (ht)
  - Multiphysics
  - Mesh 1
- Study 1
- Results

Settings Properties

Electric Potential

Label: Electric Potential 1

Boundary Selection

Selection: Manual

Active

Override and Contribution

Equation

Show equation assuming:

Study 1, Time Dependent

$V = V_0$

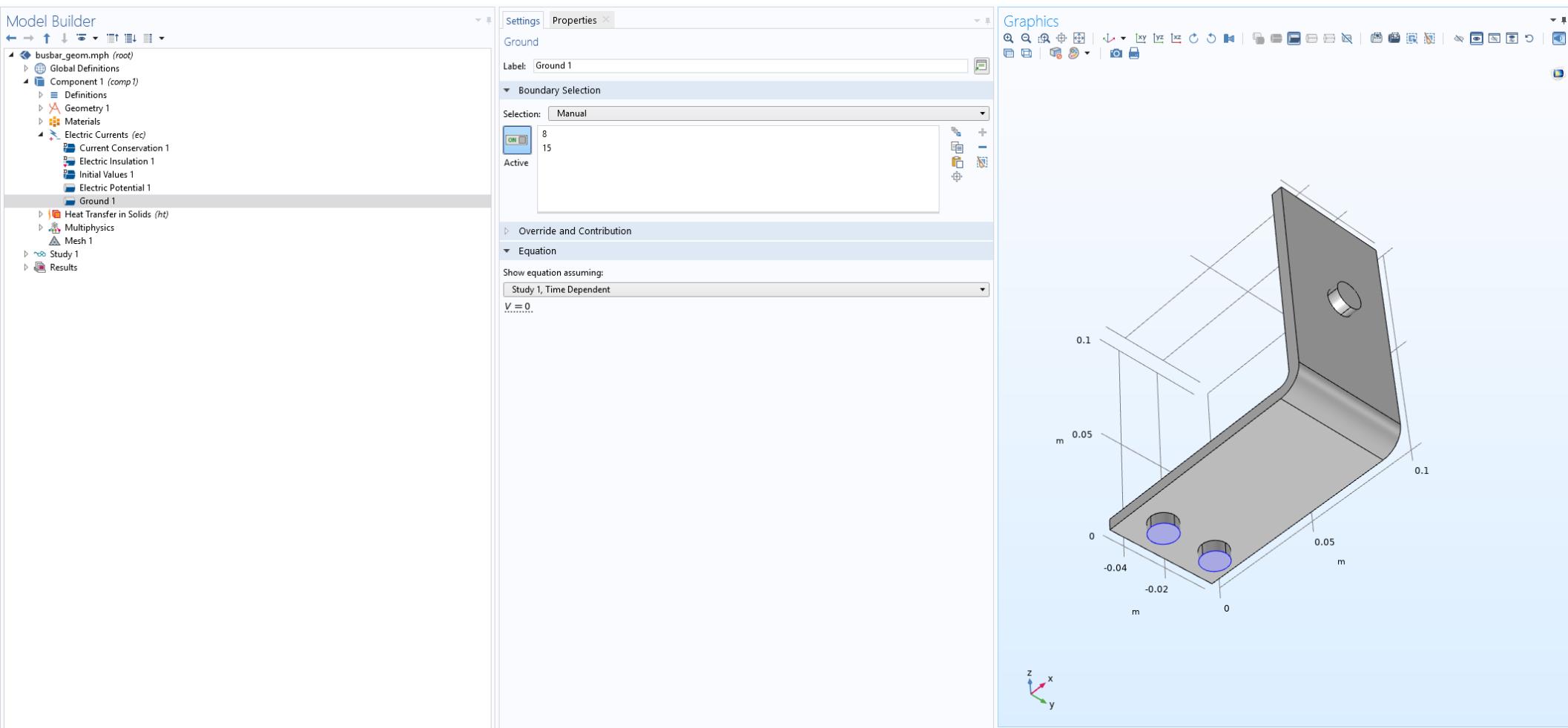
Electric Potential

Electric potential:

$V_0$  root.Vtot

Graphics

# boundary conditions (link with environment)



# boundary conditions (link with environment)

Model Builder

busbar\_geom.mph (root)

- Global Definitions
- Component 1 (comp 1)
  - Definitions
  - Geometry 1
  - Materials
  - Electric Currents (ec)
  - Heat Transfer in Solids (ht)
  - Multiphysics
  - Mesh 1
- Study 1
- Results

Properties

Heat Transfer in Solids

Label: Heat Transfer in Solids

Name: ht

Domain Selection

Selection: All domains

Active

Equation

Equation form:

Study controlled

Show equation assuming:

Study 1, Time Dependent

$\rho C_p \mathbf{u} \cdot \nabla T + \nabla \cdot \mathbf{q} = Q + Q_{\text{ted}}$

$\mathbf{q} = -k \nabla T$

Physical Model

Reference temperature:

$T_{\text{ref}}$  User defined  
293.15[K]

Heat transfer in biological tissue

Isothermal domain

Heat transfer in alloys

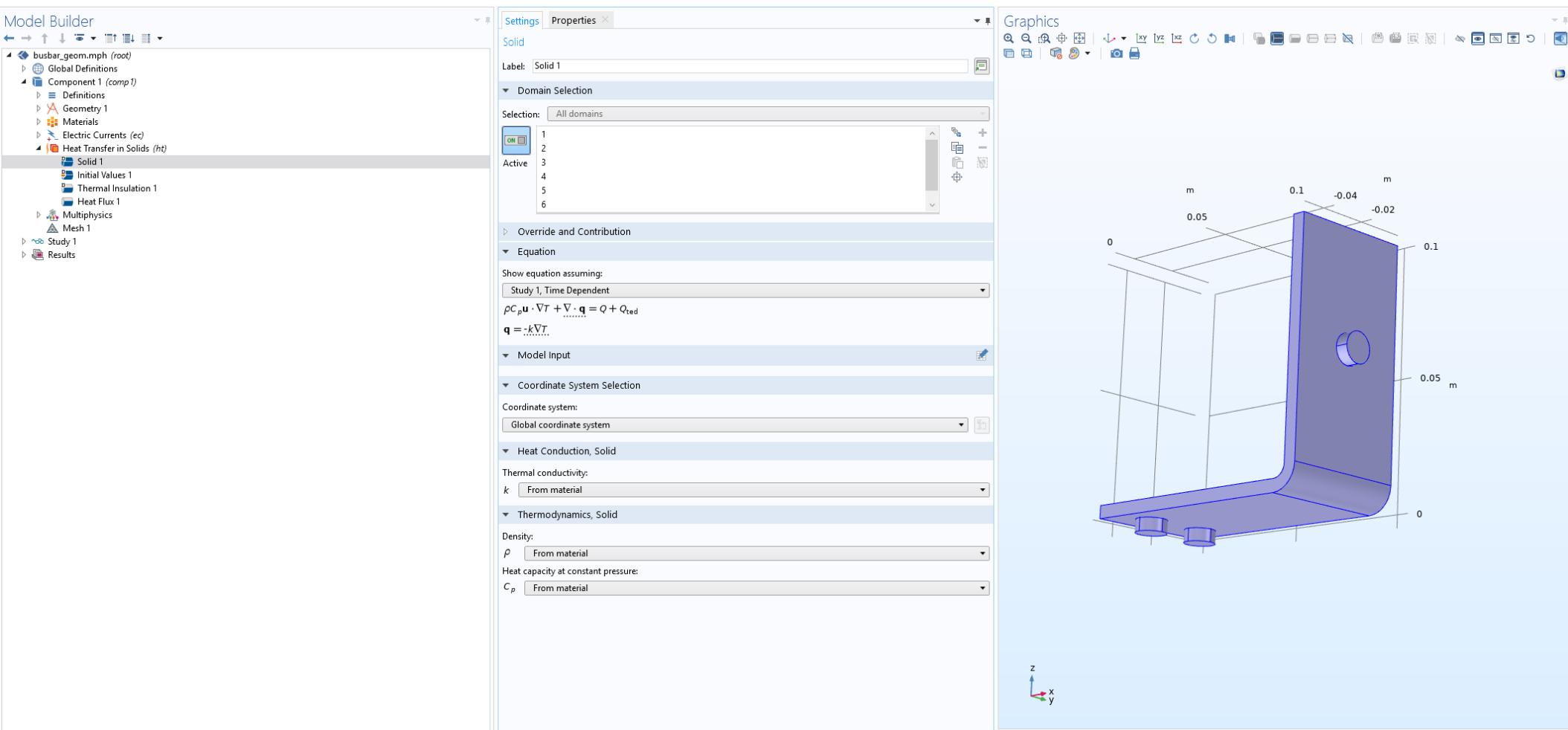
Heat transfer in porous media

Discretization

Dependent Variables

Graphics

# Laws of Physics



Model Builder

busbar.geom.mph (root)

- Global Definitions
- Component 1 (comp1)
  - Definitions
  - Geometry 1
  - Materials
  - Electric Currents (ec)
  - Heat Transfer in Solids (ht)
    - Solid 1
    - Initial Values 1
    - Thermal Insulation 1
    - Heat Flux 1
  - Multiphysics
  - Mesh 1
- Study 1
- Results

Settings Properties

Initial Values

Label: Initial Values 1

Domain Selection

Selection: All domains

Active

- ON [ ] 1
- 2
- 3
- 4
- 5
- 6

Override and Contribution

Initial Values

Temperature:

T User defined 293.15[K]

Graphics

The Graphics window displays a 3D model of a busbar component. The model is a blue-shaded solid with various dimensions labeled in meters (m). The top surface has a height of 0.1 m, a central vertical section of 0.05 m, and side sections of -0.04 m and -0.02 m. The bottom surface has a central section of 0.05 m and side sections of 0.05 m. A coordinate system (x, y, z) is shown at the bottom left.

# initial values

Model Builder

busbar\_geom.mph (root)

- Global Definitions
- Component 1 (comp 1)
  - Definitions
  - Geometry 1
  - Materials
  - Electric Currents (ec)
  - Heat Flux in Solids (ht)
    - Solid 1
    - Initial Values 1
    - Thermal Insulation 1
    - Heat Flux 1
  - Multiphysics
  - Mesh 1
- Study 1
- Results

Settings Properties

Heat Flux

Label: Heat Flux 1

Boundary Selection

Selection: All boundaries

Active

Override and Contribution

Equation

Show equation assuming: Study 1, Time Dependent

$$-\mathbf{n} \cdot \mathbf{q} = q_0$$

$$q_0 = h(T_{ext} - T)$$

Material Type

Material type: Nonsolid

Heat Flux

General inward heat flux

Convective heat flux

$q_0 = h \cdot (T_{ext} - T)$

Heat transfer coefficient:

User defined

Heat transfer coefficient:

$h = root.htc$  W/(m<sup>2</sup>·K)

External temperature:

$T_{ext}$  User defined

293.15[K]

Heat rate

$q_0 = \frac{P_0}{A}$

Graphics

The graphics window shows a 3D model of a busbar component. The model is a rectangular block with a U-shaped cutout at the bottom. The top surface has several dimensions labeled: 0.1 m (width), 0.05 m (height of the main body), and 0.05 m (height of the side wall). The U-shaped cutout has dimensions: 0.1 m (width), 0.05 m (depth), and 0.02 m (height). The front face of the main body has a width of 0.1 m and a height of 0.05 m. A coordinate system (x, y, z) is shown at the bottom left corner.

# boundary conditions (link with environment)

**Model Builder**

- busbar\_geom.mph (root)
  - Global Definitions
  - Component 1 (comp1)
    - Definitions
    - Geometry 1
    - Materials
    - Electric Currents (ec)
    - Heat Transfer in Solids (ht)
  - Multiphysics
    - Electromagnetic Heating 1 (emh1)
- Mesh 1
- Study 1
- Results

**Properties**

**Electromagnetic Heating**

Label: Electromagnetic Heating 1  
Name: emh1

**Domain Selection**  
Selection: All domains

**Boundary Selection**  
Selection: All boundaries

**Equation**

**Coupled Interfaces**

Electromagnetic:  
Electric Currents (ec)

Heat transfer:  
Heat Transfer in Solids (ht)

**Graphics**

# Laws of Physics

**Model Builder**

- busbar.geom.mph (root)
- Global Definitions
- Component 1 (comp 1)
- Study 1
  - Step 1: Time Dependent
  - Solver Configurations
  - Results

**Settings Properties**

Time Dependent

Compute  Update Solution

Label: Time Dependent

Study Settings

Time unit: min  
Times: range(0,20,180) min

Tolerance: Physics controlled

Results While Solving

Physics and Variables Selection

Modify model configuration for study step

|                              |                                     |                  |
|------------------------------|-------------------------------------|------------------|
| Physics interface            | Solve for                           | Discretization   |
| Electric Currents (ec)       | <input checked="" type="checkbox"/> | Physics settings |
| Heat Transfer in Solids (ht) | <input checked="" type="checkbox"/> | Physics settings |

Multiphysics couplings

|                                  |           |                                     |
|----------------------------------|-----------|-------------------------------------|
| Electromagnetic Heating 1 (emh1) | Solve for | <input checked="" type="checkbox"/> |
|----------------------------------|-----------|-------------------------------------|

Values of Dependent Variables

Initial values of variables solved for

Settings: Physics controlled

Values of variables not solved for

Settings: Physics controlled

Store fields in output

Settings: All

Mesh Selection

Geometry Mesh

Geometry 1 Mesh 1

Study Extensions

Auxiliary sweep

Sweep type: Specified combinations

| Parameter name | Parameter value list | Parameter unit |
|----------------|----------------------|----------------|
|                |                      |                |

Adaptive mesh refinement

Adaptation in geometry: Geometry 1

Automatic remeshing

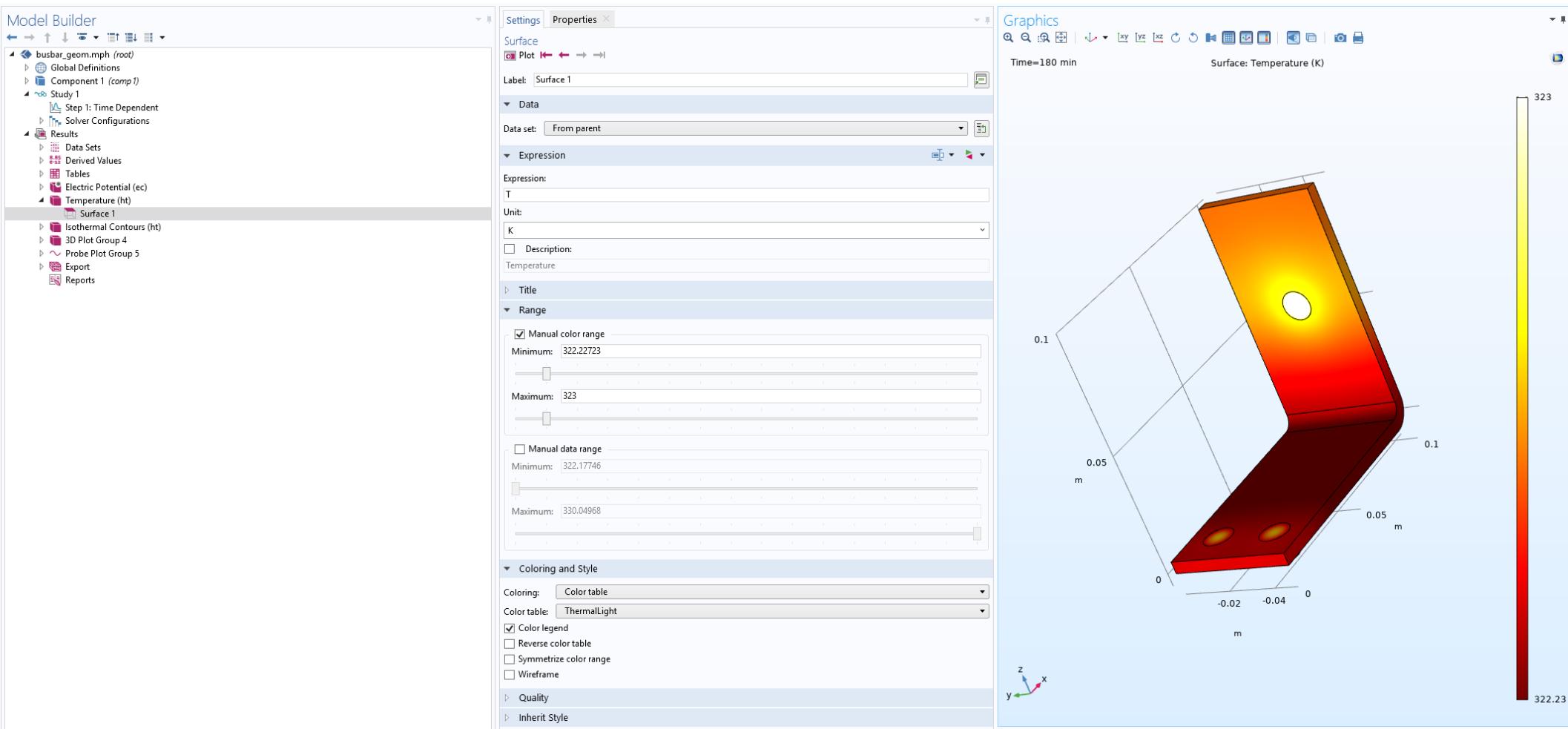
Remesh in geometry: Geometry 1

**Graphics**

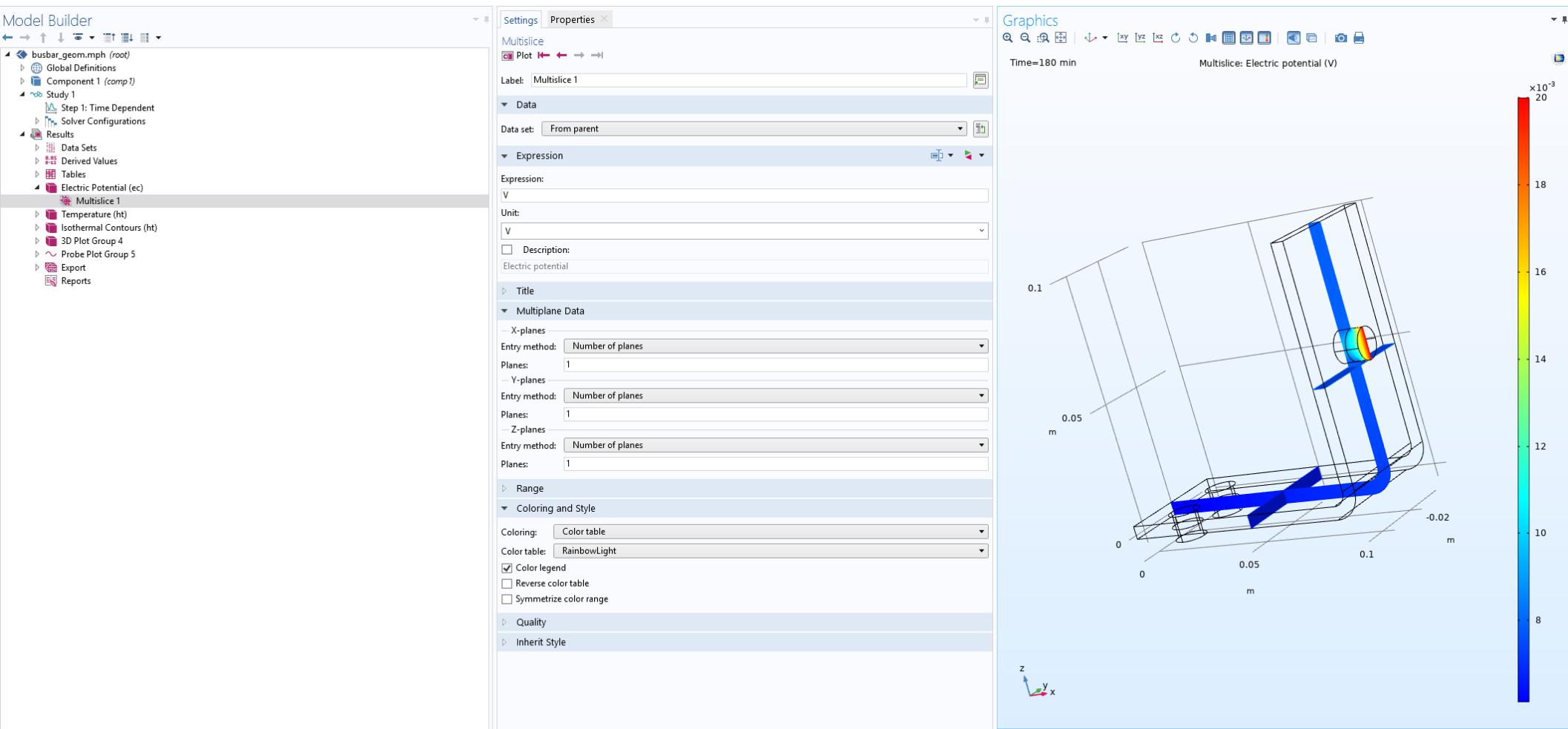
Messages Progress Log Objective Table 2

COMSOL Multiphysics 5.40.388  
License will expire in 2 days.  
[Oct 16, 2019 11:02 AM] Opened file: C:\Users\Simon\Documents\Unief\18.PostDoc\IOF-SBO.Frames\19.10.03.ComsolTutorial\Exercise1.mph

# experiment



# experiment result (temperature distribution)

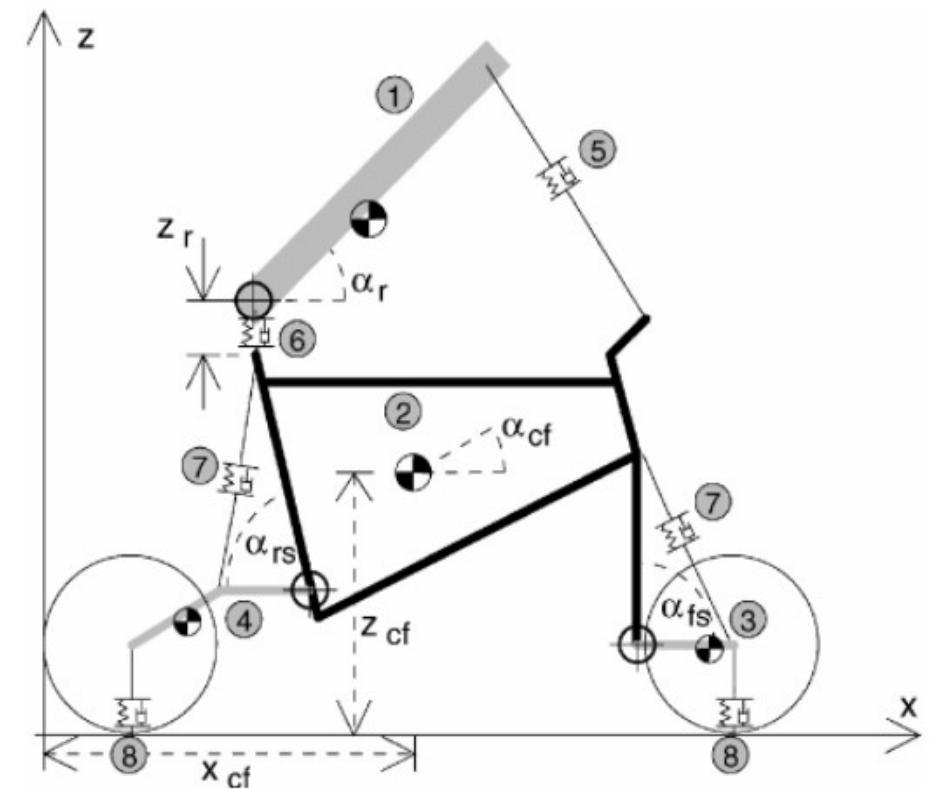


# experiment result (voltage distribution)

# Parameters

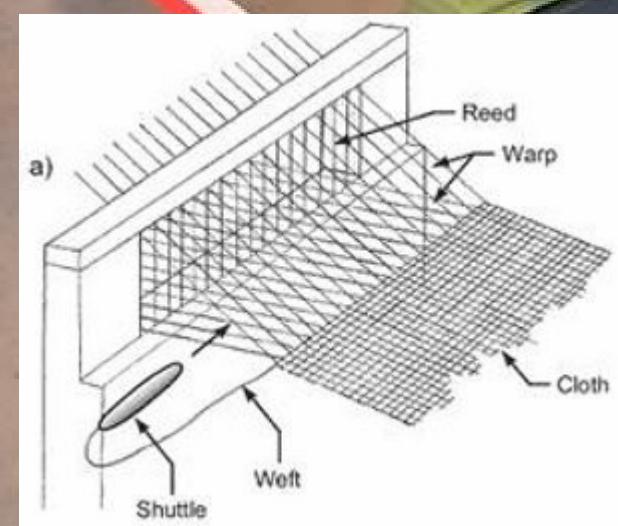


Distributed

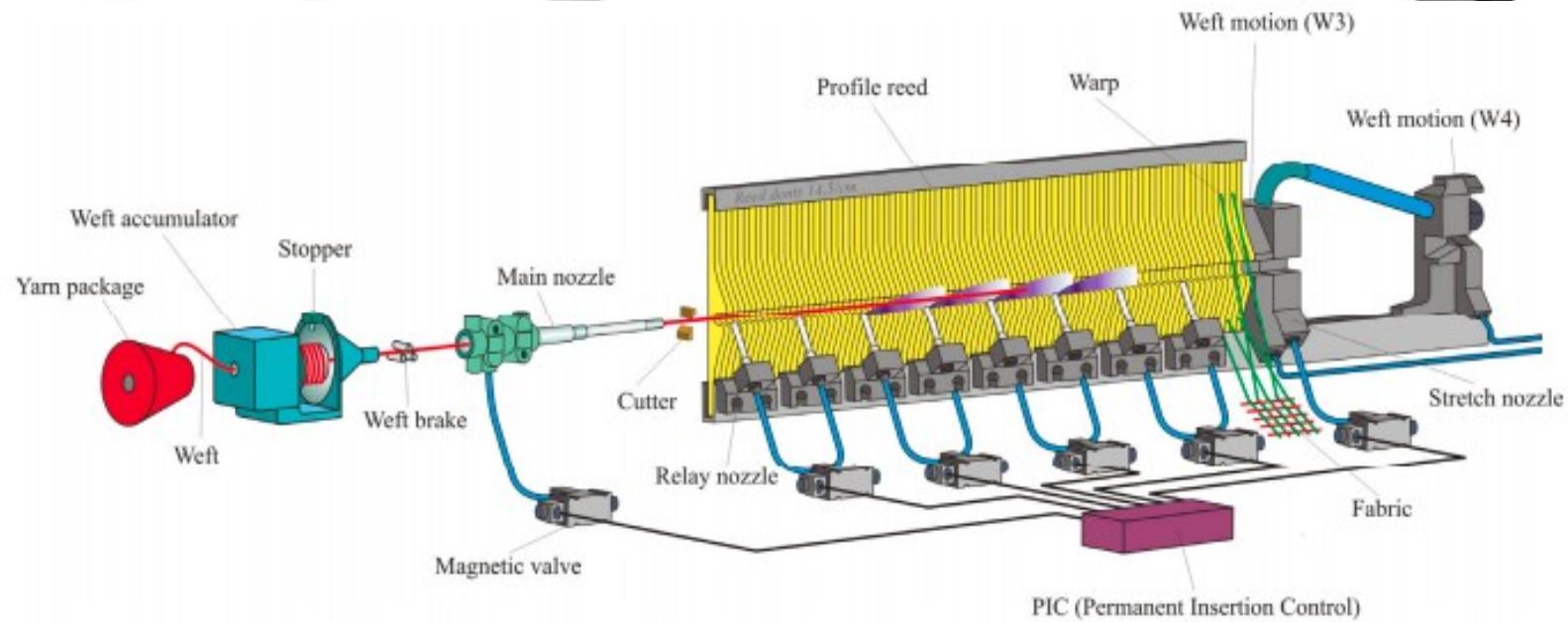


Lumped

# Shuttle Loom



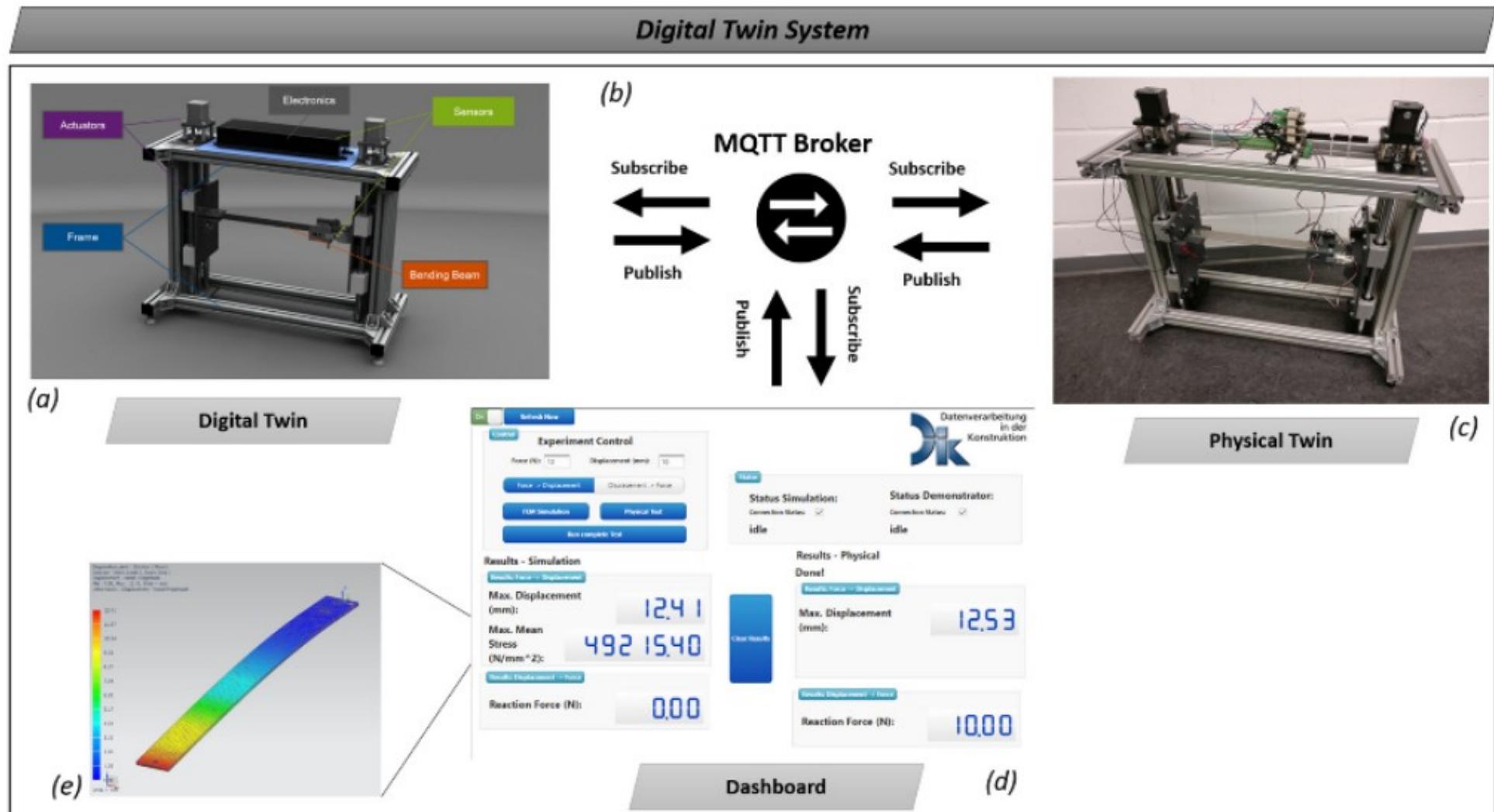
# Air Jet Loom



# distributed + lumped parameter models

S. Haag, R. Anderl / Manufacturing Letters 15 (2018) 64–66

65



**Fig. 1.** Overview of the Digital Twin System. The Digital Twin (a) and the Physical Twin (c) are connected through a broker-client-architecture (b). The system is controlled via a web-based dashboard (d) accessible from any internet-capable device. Running a complete test through the dashboard will trigger the actuators of the physical twin as well as a FEM simulation. Results are shown numerically on the dashboard as well as graphically in the CAD system (e).

# generative design (Design-Space Exploration – DSE)



**SIEMENS**  
*Ingenuity for life*

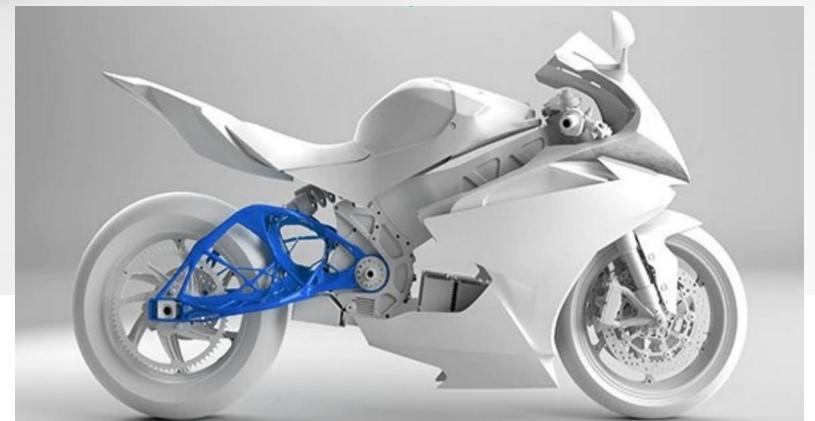


**The World's Fastest Production  
Electric Motorcycle - Lightning LS-218**

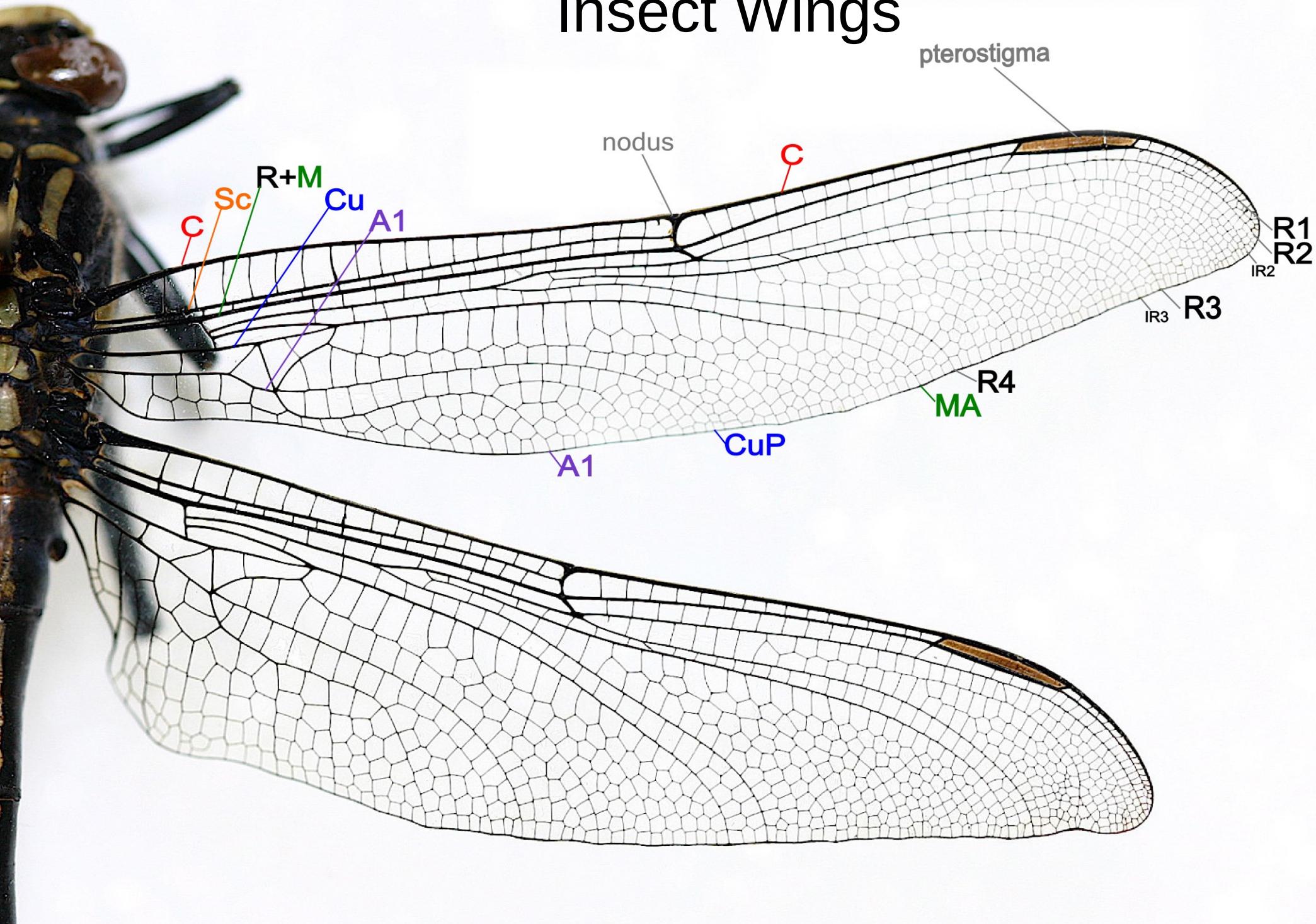


 **AUTODESK**  
Make anything.

<https://www.autodesk.com/solutions/generative-design>



# Insect Wings

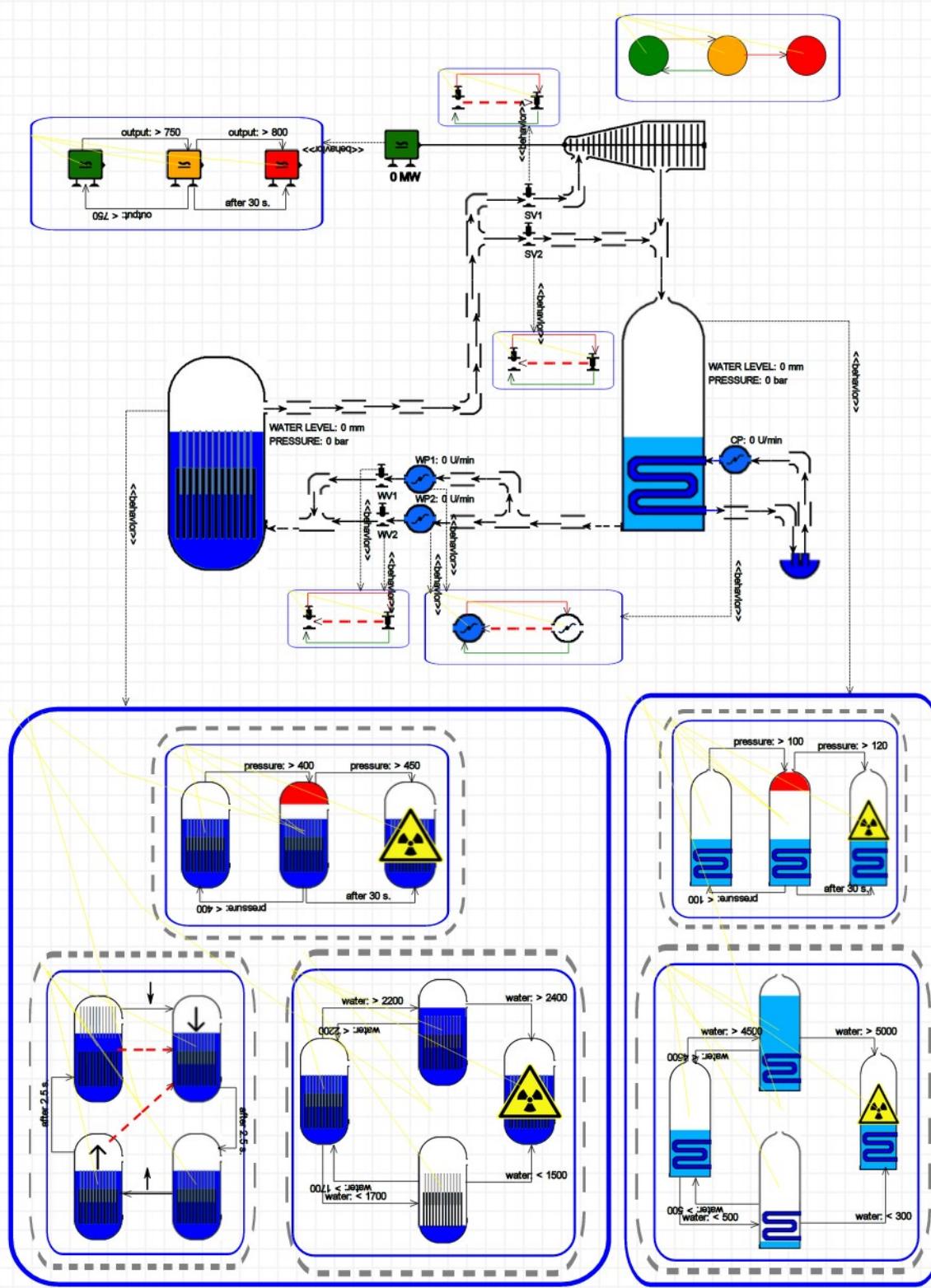


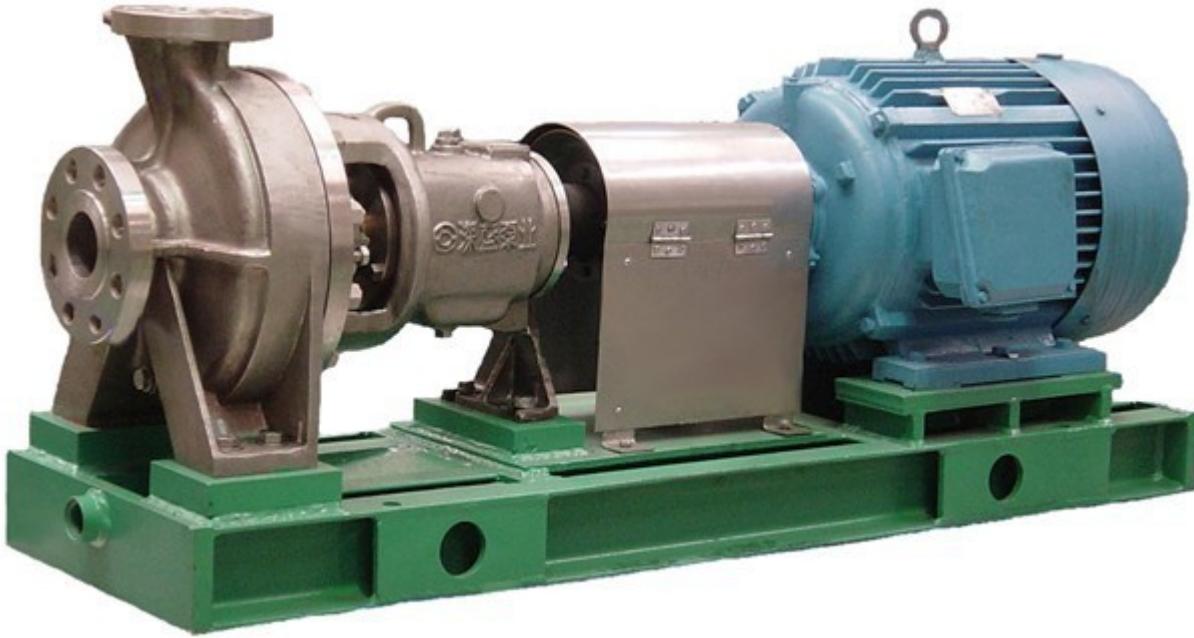
# Physical Systems Modelling

- Problem-Specific (technological)
- Domain-Specific (e.g., translational mechanical)
- (general) Laws of Physics
- Power Flow/Bond Graphs (physical: energy/power)
- Computationally a-causal  
(Mathematical and Object-Oriented) ← **Modelica**
- Causal Block Diagrams (data flow)
- Numerical (Discrete) Approximations
- Computer Algorithmic + Numerical  
(Floating Point vs. Fixed Point)
- As-Fast-As-Possible vs. Real-time (XiL)
- Hybrid (discrete-continuous) modelling/simulation
- Hiding IP: Composition of Functional Mockup Units (FMI)
- Dynamic Structure

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### Boric Acid Transportation Pump

#### Product parameters

Design standards : RCC-M

Flow : 16.6m<sup>3</sup>/h

Head : 85m

Temperature : ~80°C

Pressure : 1.6MPa

Used in 600MWe、900MWe、1000MWe PWR nuclear power plant boric acid transportation system.

# Physical Systems Modelling

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# SimHydraulics

File Edit View Display Diagram Simulation Analysis Code Tools Help

sh\_actuator\_with\_2\_chamber\_snubbers\_model/Double-Acting Hydraulic Cylinder with Snubbers

Double-Acting Hydraulic Cylinder with Snubbers

sh\_actuator\_with\_2\_chamber\_snubbers\_model > Double-Acting Hydraulic Cylinder with Snubbers

1 C  
2 A  
3 R  
4 B

Cushioning Orifice A  
Variable Orifice A  
Check Valve A  
Main Piston A  
Motion Sensor  
Hard Stop  
Cushioning Orifice B  
Variable Orifice B  
Check Valve B  
Main Piston B  
Cushion Piston A  
Cushion Piston B

Velocity /Position

Pressure

Time offset: 0

Diagram showing a double-acting hydraulic cylinder model with two chambers and snubbers. The cylinder has four ports: C (top inlet), A (bottom inlet), R (top exhaust), and B (bottom exhaust). The circuit includes two main pistons (A and B) connected to cushion pistons (A and B) via check valves (A and B). Motion sensors and variable orifices are used for damping and cushioning. A hard stop is also present. Three plots show Velocity/Position and Pressure over time.

3D CAD model of the cylinder assembly, showing the internal piston and rod components.

## Why DS(V)M ? (as opposed to General Purpose modelling)

- **match the user's mental model** of the problem domain
- **maximally constrain** the user (to the problem at hand)
  - ⇒ easier to learn
  - ⇒ avoid errors
- **separate** domain-expert's work from analysis/transformation expert's work

### Anecdotal evidence of 5 to 10 times speedup

Steven Kelly and Juha-Pekka Tolvanen. Domain-Specific Modeling: Enabling Full Code Generation. Wiley, 2008.

Laurent Safa. The practice of deploying DSM, report from a Japanese appliance maker trenches. In Proceedings of the 6th OOPSLA Workshop on Domain-Specific Modeling (DSM'06), pp. 185-196, 2006.

++ more potential for optimization thanks to more (tighter) “type” information       $V^2 = 4$



In der Beschränkung zeigt sich erst der Meister.  
(Johann Wolfgang von Goethe)

# Physical Systems Modelling

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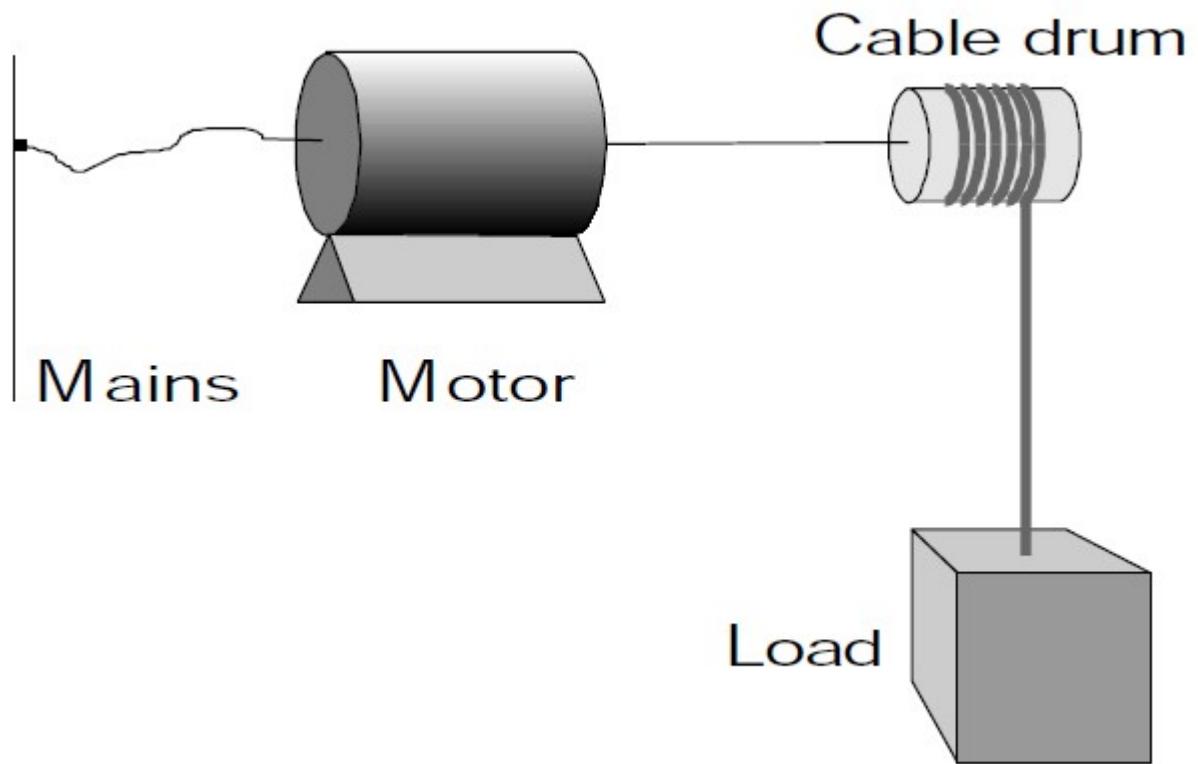
Paulo Carreira · Vasco Amaral · Hans Vangheluwe  
*Editors*

# Foundations of Multi-Paradigm Modelling for Cyber-Physical Systems

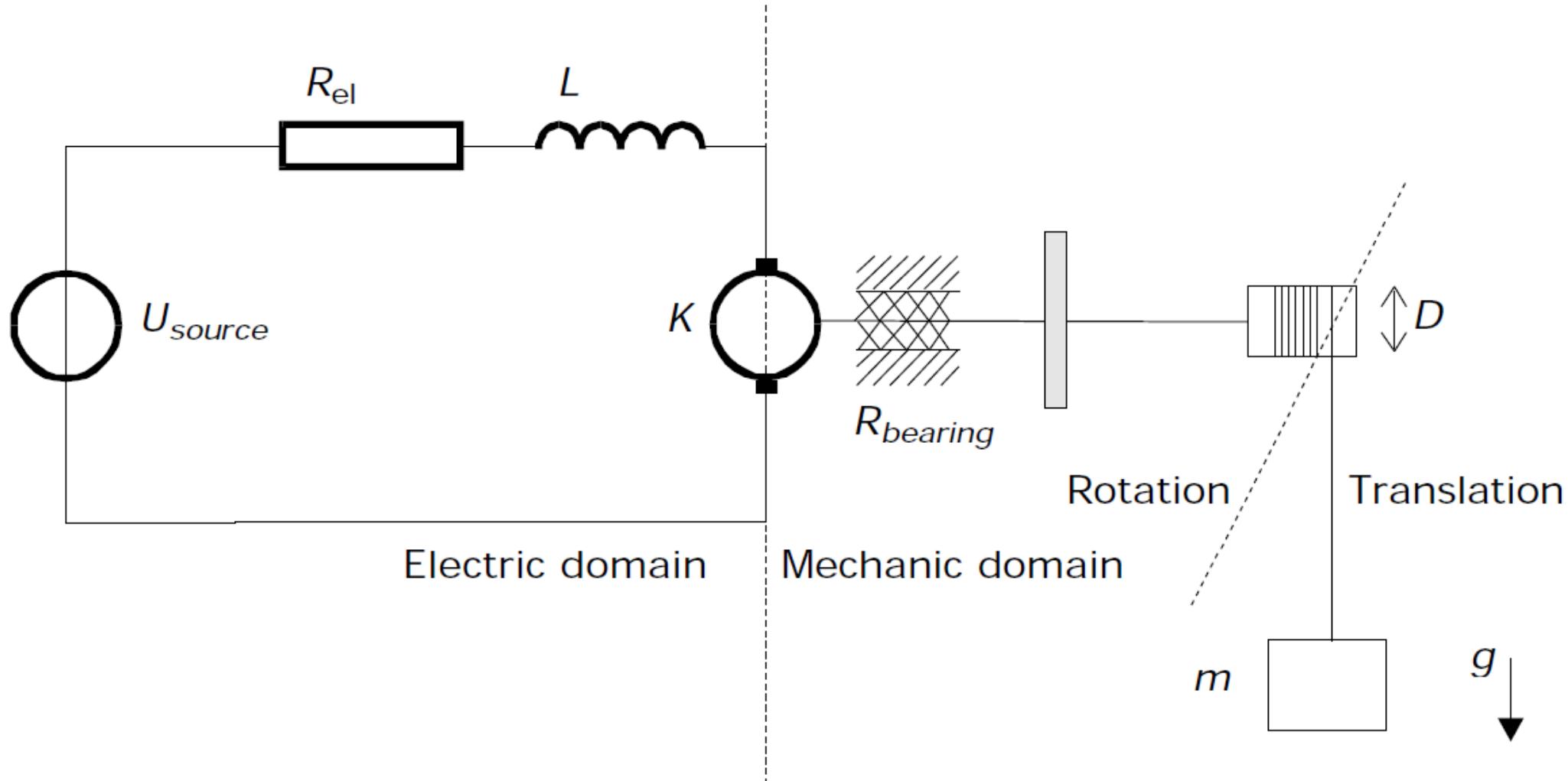


Broenink J.F. (2020) Bond Graphs: A Unifying Framework for Modelling of Physical Systems.  
In: Carreira P., Amaral V., Vangheluwe H. (eds) Foundations of Multi-Paradigm Modelling for Cyber-Physical Systems. Springer, Cham.  
[https://doi.org/10.1007/978-3-030-43946-0\\_2](https://doi.org/10.1007/978-3-030-43946-0_2)

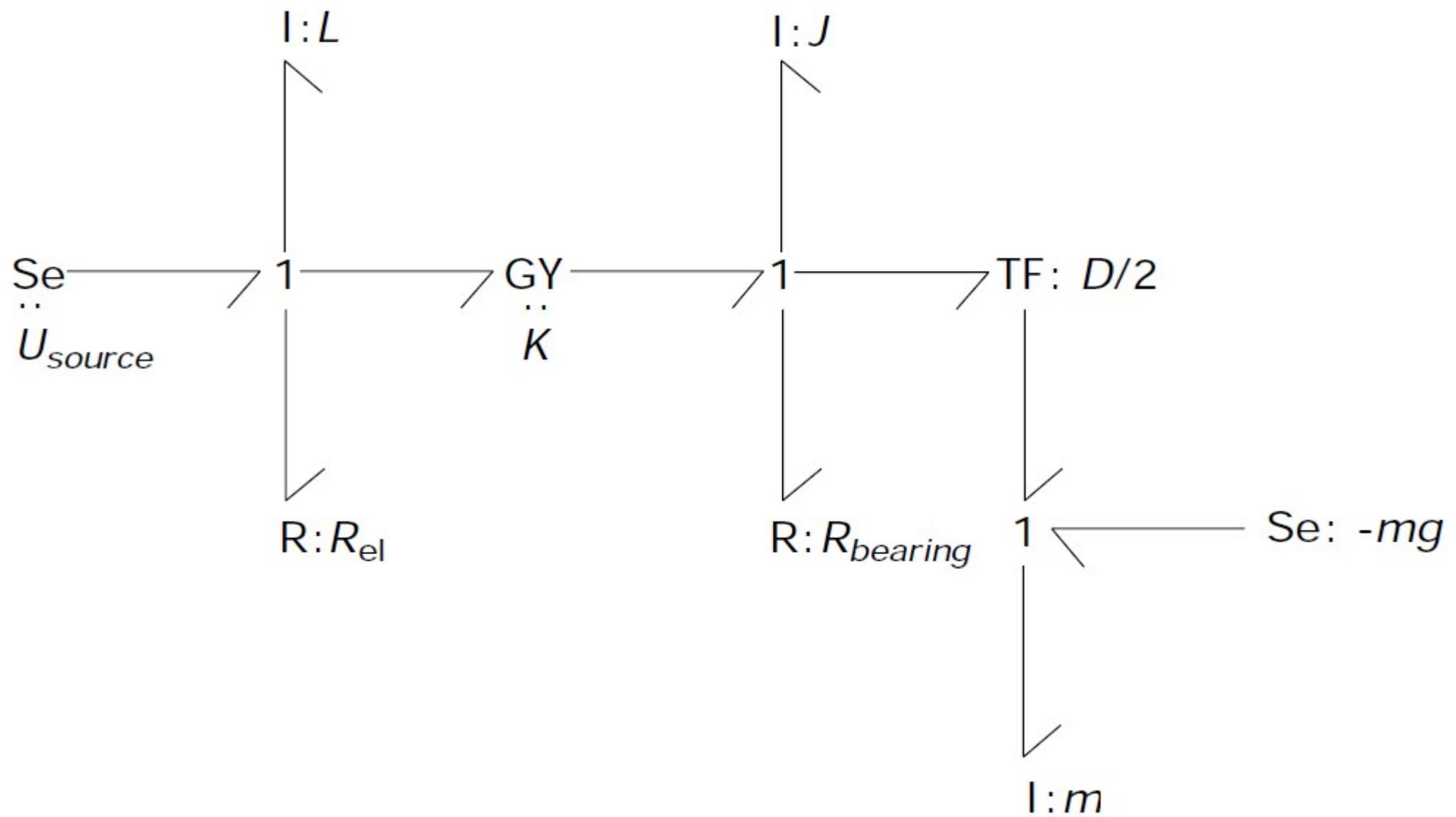
model using domain notation



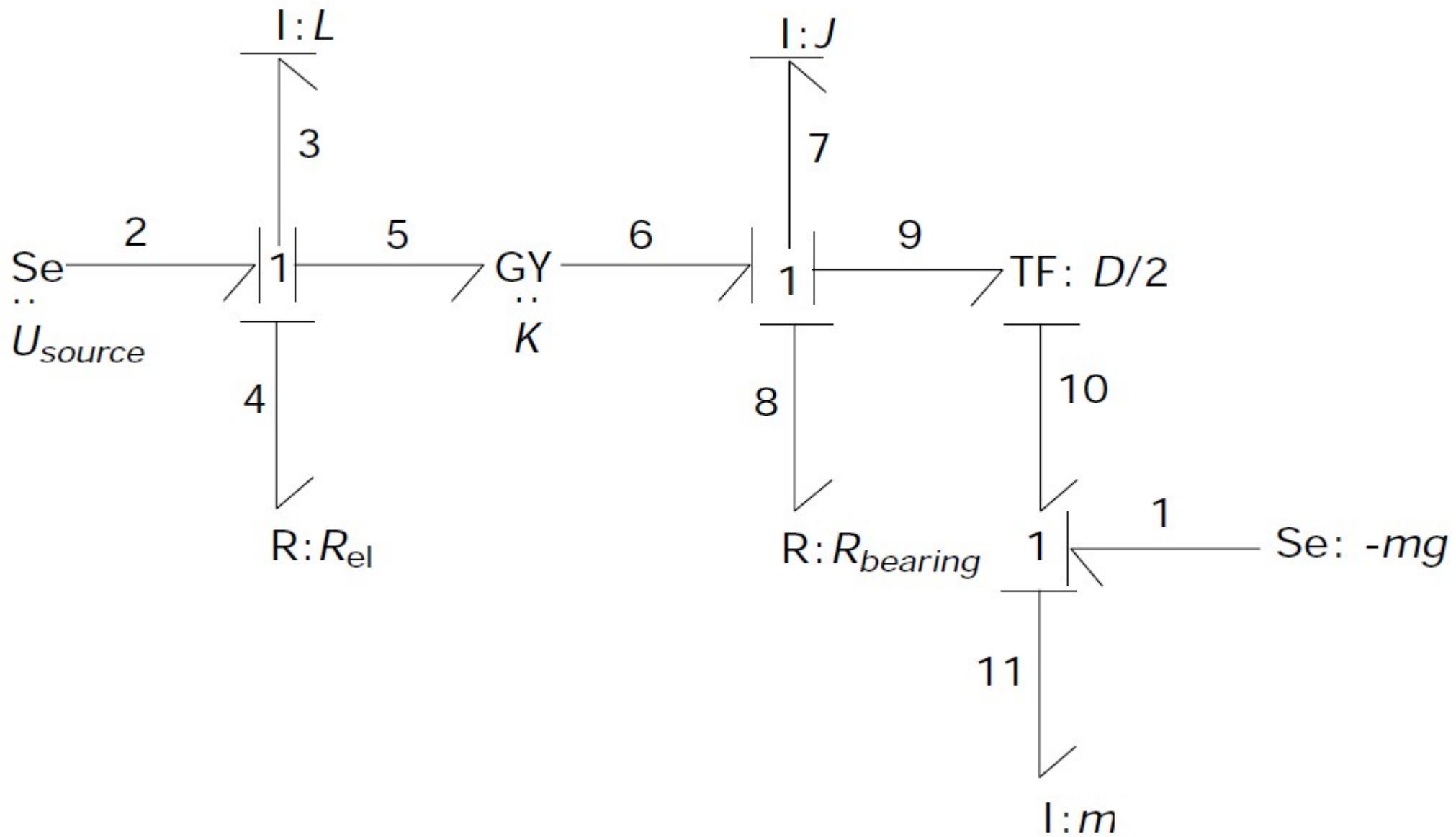
# Idealized Physical Model (IPM) 1D aka “lumped parameter”



a-causal

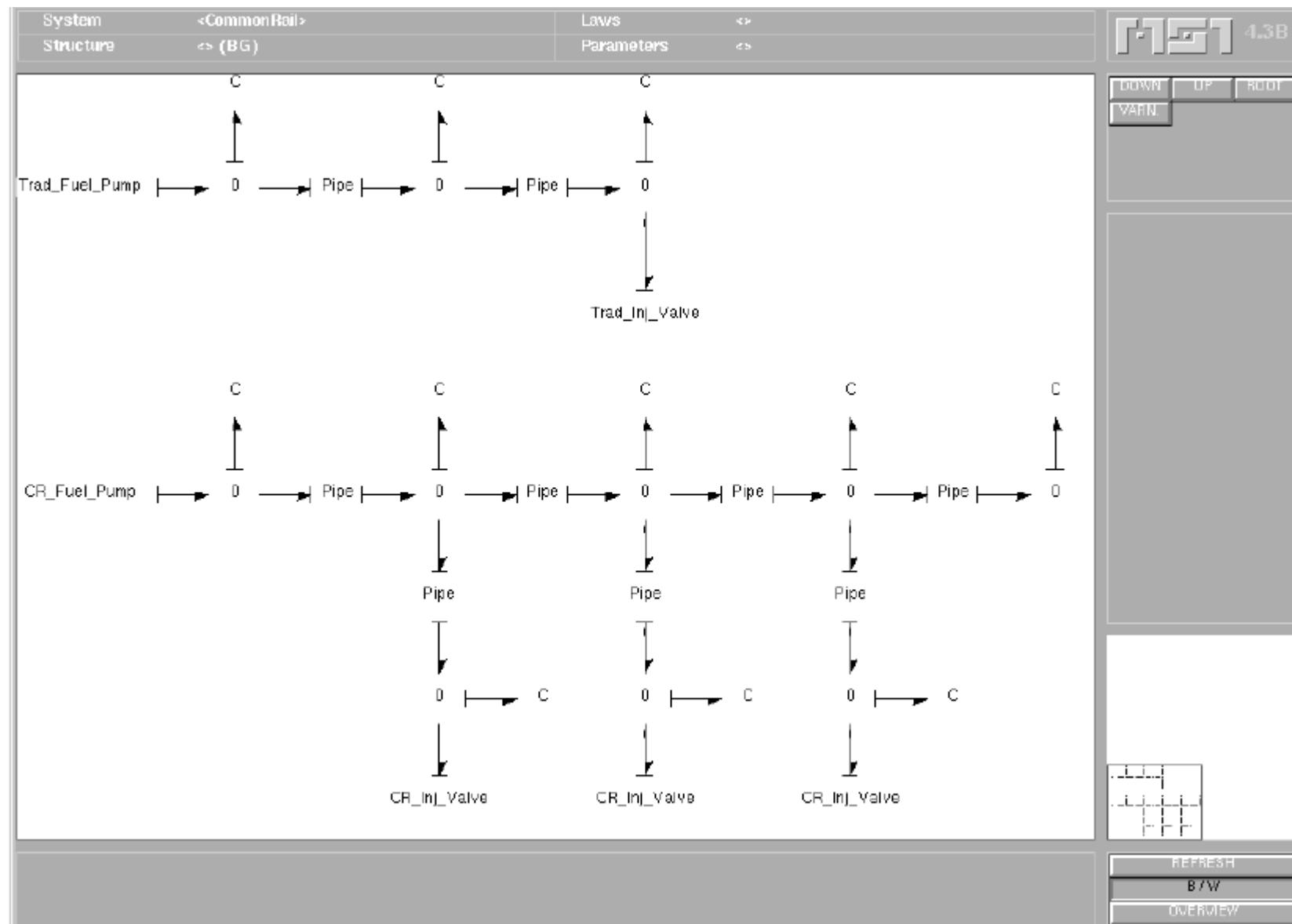


## Causal (after “causality assignment” – propagation)

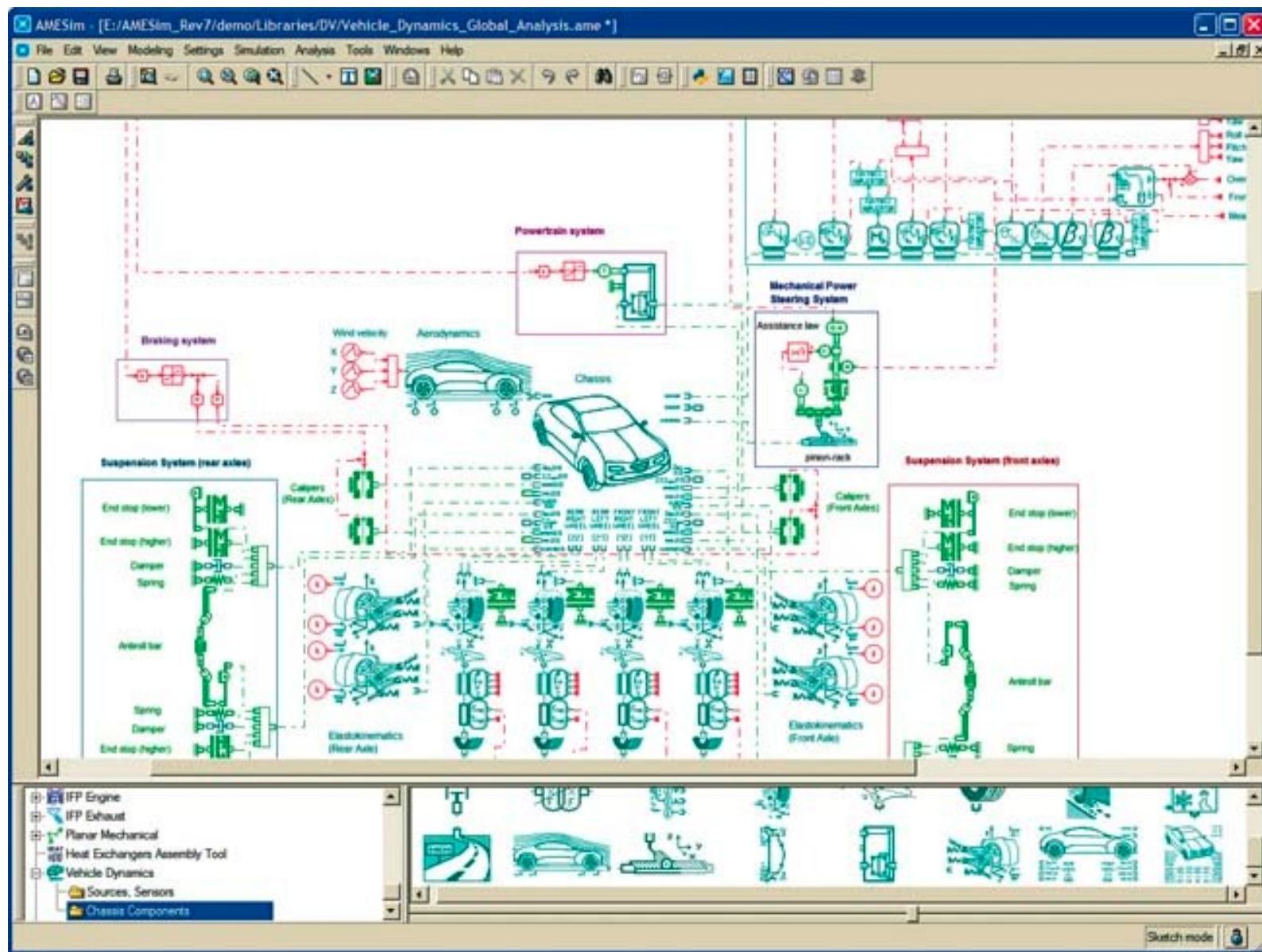


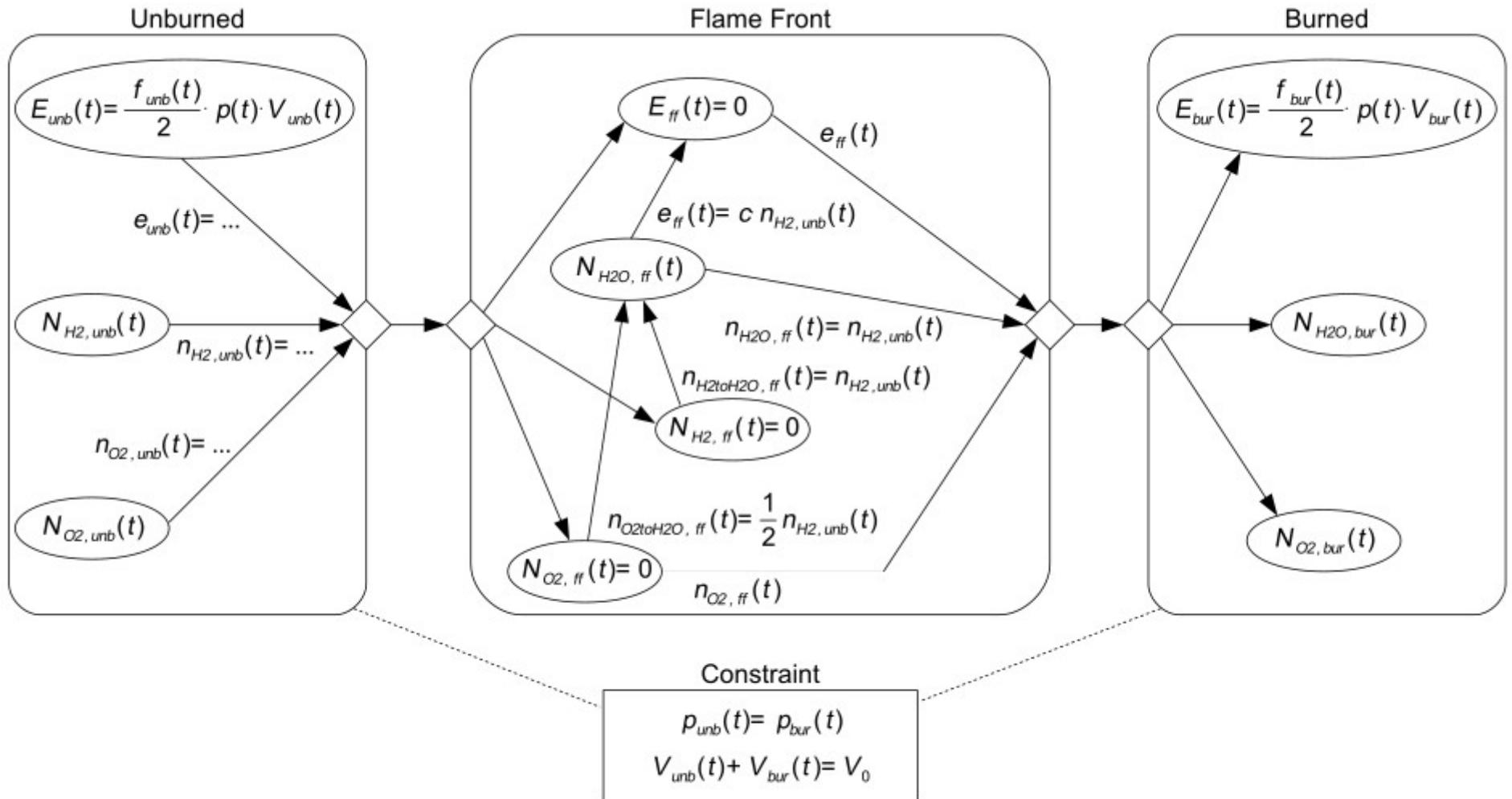
|                               | $f$<br>flow                  | $E$<br>effort         | $q = \int f dt$<br><i>generalized displacement</i> | $p = \int e dt$<br><i>generalized momentum</i>  |
|-------------------------------|------------------------------|-----------------------|--|---|
| <i>Electromagnetic</i>        | $i$<br>current               | $U$<br>voltage        | $q = \int i dt$<br>charge                          | $\lambda = \int u dt$<br>magnetic flux linkage  |
| <i>mechanical translation</i> | $V$<br>velocity              | $F$<br>force          | $x = \int v dt$<br>displacement                    | $p = \int F dt$<br>momentum                     |
| <i>mechanical rotation</i>    | $\omega$<br>angular velocity | $T$<br>torque         | $\theta = \int \omega dt$<br>angular displacement  | $b = \int T dt$<br>angular momentum             |
| <i>hydraulic/ pneumatic</i>   | $\varphi$<br>volume flow     | $P$<br>pressure       | $V = \int \varphi dt$<br>volume                    | $\Gamma = \int p dt$<br>momentum of a flow tube |
| <i>Thermal</i>                | $T$<br>temperature           | $F_S$<br>entropy flow | $S = \int f_S dt$<br>entropy                       |   |
| <i>Chemical</i>               | $\mu$<br>chemical potential  | $F_N$<br>molar flow   | $N = \int f_N dt$<br>number of moles               |   |

| mechanical rotation                  | mechanical translation | electrical           | hydraulic                             |
|--------------------------------------|------------------------|----------------------|---------------------------------------|
|                                      |                        |                      |                                       |
| spring                               | spring                 | condensor            | reservoir                             |
|                                      |                        |                      |                                       |
| inertia                              | mass                   | coil                 | hydraulic inertia                     |
|                                      |                        |                      |                                       |
| damper                               | damper                 | resistor             | (flow)<br>resistance                  |
|                                      |                        |                      |                                       |
| friction                             | friction               | variable<br>resistor | valve                                 |
|                                      |                        |                      |                                       |
| torque source                        | force source           | voltage source       | pressure source<br>(centrifugal pump) |
|                                      |                        |                      |                                       |
| angular velocity<br>source<br>source | velocity source        | current source       | flow source<br>(displacement pump)    |
|                                      |                        |                      |                                       |
| gear box<br>(transformer)            | lever<br>(transformer) | transformer          | pressure amplifier                    |



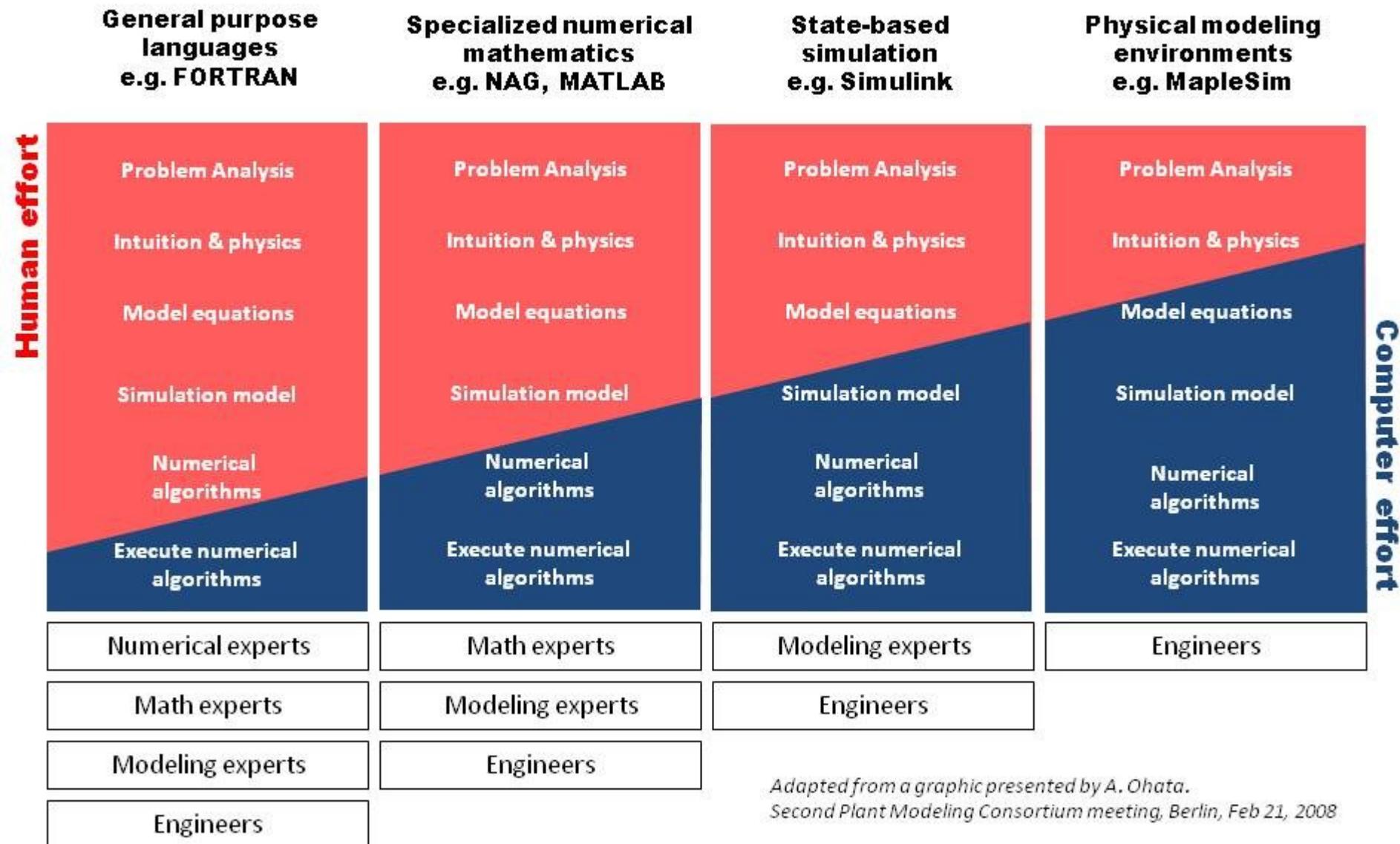
# Imagine.Lab AMESim





**Figure 2.** High Level Model Description (HLMD) example - hydrogen-oxygen combustion in a closed chamber.

Akira Ohata @ Toyota

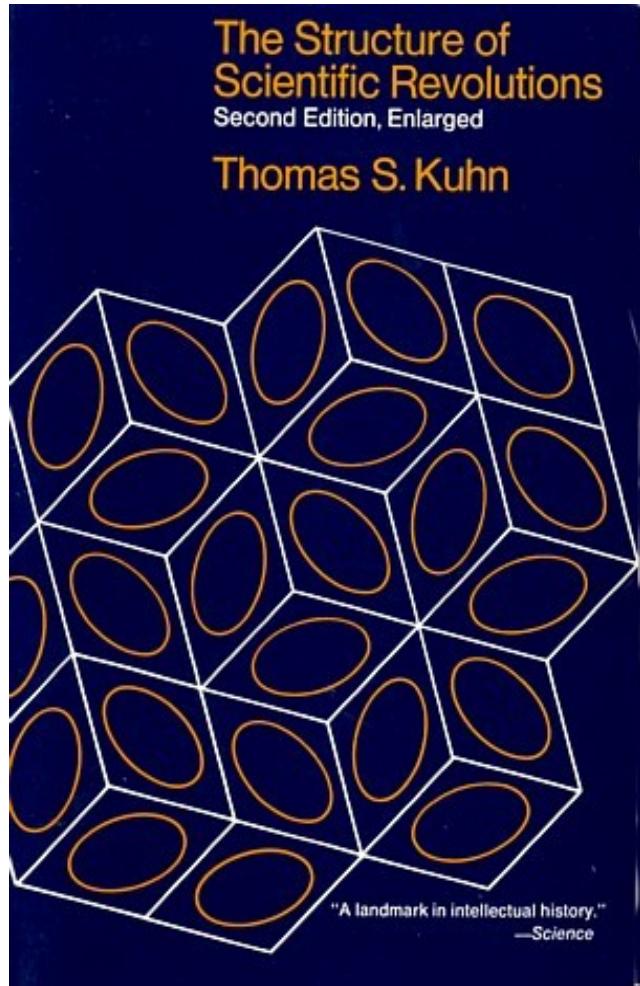


*Adapted from a graphic presented by A. Ohata.  
Second Plant Modeling Consortium meeting, Berlin, Feb 21, 2008*

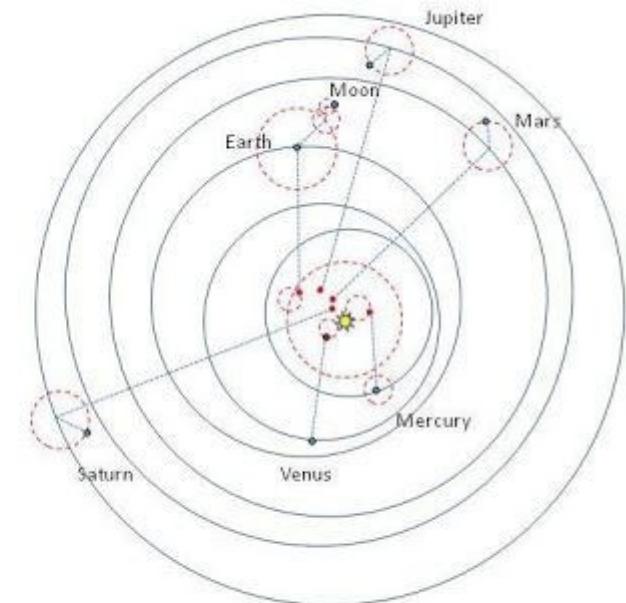
# Physical Systems Modelling

- Problem-Specific (technological)
- Domain-Specific (e.g., translational mechanical)
- (general) Laws of Physics
- Power Flow/Bond Graphs (physical: energy/power)
- Computationally a-causal  
(Mathematical and Object-Oriented) ← **Modelica**
- Causal Block Diagrams (data flow)
- Numerical (Discrete) Approximations
- Computer Algorithmic + Numerical  
(Floating Point vs. Fixed Point)
- As-Fast-As-Possible vs. Real-time (XiL)
- Hybrid (discrete-continuous) modelling/simulation
- Hiding IP: Composition of Functional Mockup Units (FMI)
- Dynamic Structure

# Thomas Kuhn: “paradigm shift”



Ptolemaic Model



Copernican Model

Paulo Carreira · Vasco Amaral · Hans Vangheluwe  
*Editors*

# Foundations of Multi-Paradigm Modelling for Cyber-Physical Systems



cost  
EUROPEAN COOPERATION  
IN SCIENCE & TECHNOLOGY



Springer Open

Fritzson P. (2020) Modelica: Equation-Based, Object-Oriented Modelling of Physical Systems.  
In: Carreira P., Amaral V., Vangheluwe H. (eds) Foundations of Multi-Paradigm Modelling for Cyber-Physical Systems. Springer, Cham.  
[https://doi.org/10.1007/978-3-030-43946-0\\_3](https://doi.org/10.1007/978-3-030-43946-0_3)

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Författare  
00 Hilding Elmquist

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REPORT LUTFD2/(TFRT-1015)/1-226/(1978)  
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May 1978  
Årendebeteckning  
0676



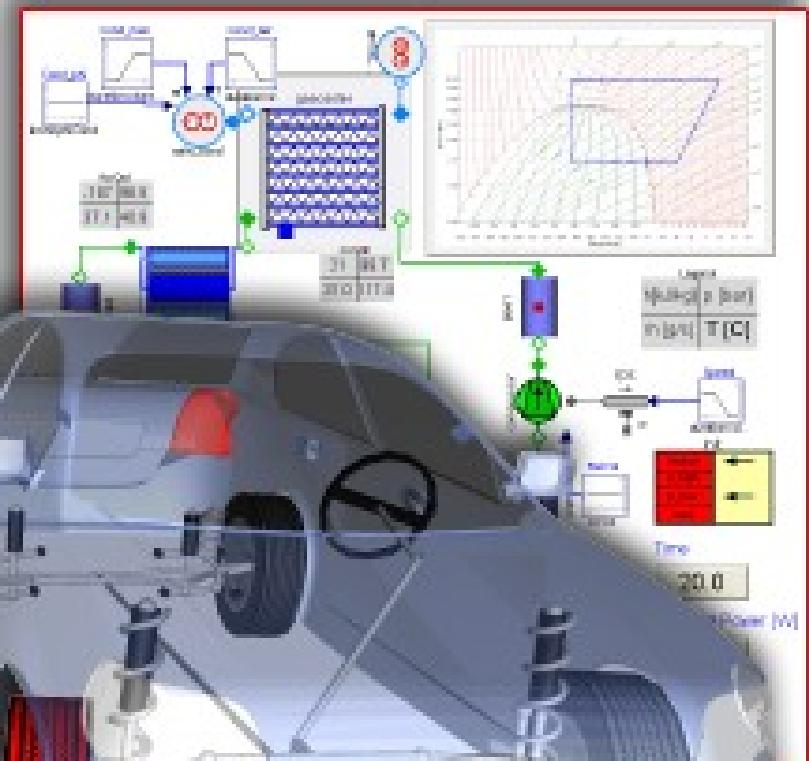
|   |   |
|---|---|
| Dokumenttitel och undertitel<br>1810<br>A Structured Model Language for Large Continuous Systems  |   |
| Referat (sammendrag)<br>2010<br>A model language, called DYMOLA, for continuous dynamical systems is proposed. Large models are conveniently described hierarchically using a submodel concept. The ordinary differential equations and algebraic equations need not be converted to assignment statements. There is a concept, cut, which corresponds to connection mechanisms of complex types, and there are facilities to describe the connection structure of a system. A model can be manipulated for different purposes such as simulation and static calculations. The model equations are sorted and they are converted to assignment statements using formula manipulation. A translator for the model language is also included. |   |
| Referat skrivet av<br>42 Author   |   |
| Förslag till ytterligare nyckelord<br>6011<br>nonlinear systems, compiler, permutations, graph theory   |   |
| Klassifikationssystem och -klasser<br>5070  |   |
| Indexterminer (långa källa)<br>52 Mathematical models, Simulation languages, Computerized simulation, Nonlinear systems, Ordinary differential equations, Compilers.<br>(Thesaurus of Engineering and Scientific Terms, Eng. Joint Council, USA)  |   |
| Omfång<br>5022 226 pages  | Övriga bibliografiska uppgifter<br>5072 |
| Språk<br>50 English   |   |
| Sekretessuppgifter<br>5070  |   |
| Dokumentet kan erhållas från<br>04 Department of Automatic Control<br>Lund Institute of Technology<br>P O Box 725, S-220 07 Lund 7, Sweden  |   |
| Pris<br>5070  |   |

- Modelica
- User's Guide
- Blocks
- Mechanics
- Fluid
- Electrical
- Analog
- Examples
- Basic
  - Ground
  - Resistor
  - Conductor
  - Capacitor
  - Inductor
  - SaturatingInductor
  - DCTransformer
  - M\_Transformer
  - Gyrator

```

model Capacitor "Ideal linear electrical capacitor"
  parameter SI.Capacitance C "Capacitance";
  Interfaces.PositivePin p;
  Interfaces.NegativePin n;
  SI.Voltage v "Voltage drop between pins";
equation
  0 = p.i + n.i;
  v = p.v - n.v;
  C*der(v) = p.i;
end Capacitor;

```



# MODELICA



# Simulation in Europe



ESPRIT Basic Research Working Group 8467

Simulation for the Future: New Concepts, Tools and Applications

## Keywords:

simulation technologies, multi-paradigm modelling, solvers, standards, interoperability, industrial deployment, demonstrators, user-simulator interfaces



Modelica  
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Blocks  
Mechanics  
Fluid  
Electrical  
Optical  
Examples  
Basic  
Ground  
Resistor  
Conductor  
Capacitor  
Inductor  
SaturatingInductor  
Transformer  
M Transformer  
Gyroscope

```
model Capacitor "ideal linear electrical capacitor"
  parameter SI.Capacitance C "Capacitance";
  Interfaces.PositivePin p;
  Interfaces.NegativePin n;
  SI.Voltage v "Voltage drop between pins";
  equations
    C * der(v) = p;
    v = p - n;
  end Capacitor;
```

M O D E L I C A

OpenModelica

# Equation-Based Object-Oriented Modeling Languages and Tools

| [home](#) | [EOOLT 2017](#) |

## News

### [EOOLT 2017](#)

The EOOLT workshop took successfully place in Munich, Germany on December 1.

Proceedings are now available on ACM Digital Library

### **Modelica Scalable Test Suite**

A new suite of scalable test models [can be found here](#).

## Welcome to the EOOLT community!

This site is intended to be a meeting point for researchers and practitioners working in the area of equation-based object-oriented modeling languages and tools. The site's main purpose is to host the workshop pages for the EOOLT workshop series. Below you can find links to the current and past events, together with links to the open access workshop proceedings.

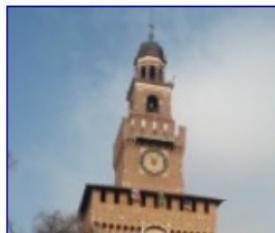
This site is maintained by [David Broman](#). If you have any questions or comments, please send an [email](#).



**EOOLT 2017, December 1, Munich, Germany**  
8th International Workshop on Equation-Based Object-Oriented Modeling Languages and Tools

[EOOLT 2017 Proceedings \(ACM Digital Library\)](#)

[Workshop site](#)



**EOOLT 2016, April 18, Milano, Italy**  
7th International Workshop on Equation-Based Object-Oriented Modeling Languages and Tools

[EOOLT 2016 Proceedings \(ACM Digital Library\)](#)

[Workshop site \(archived\)](#)



**EOOLT 2014, Berlin, Germany**  
6th International Workshop on Equation-Based Object-Oriented Modeling Languages and Tools

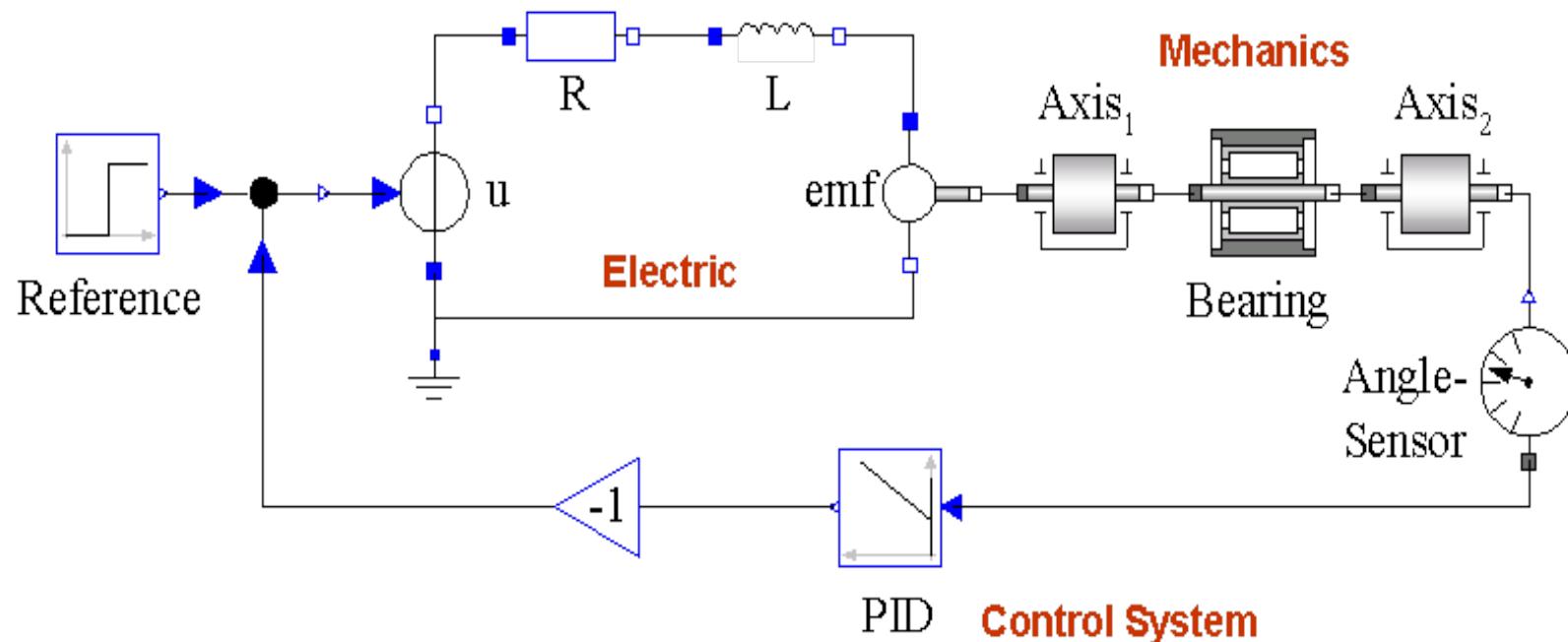
[EOOLT 2014 Proceedings \(ACM Digital Library\)](#)

[Workshop site \(archived\)](#)

# Multi-Domain Modeling

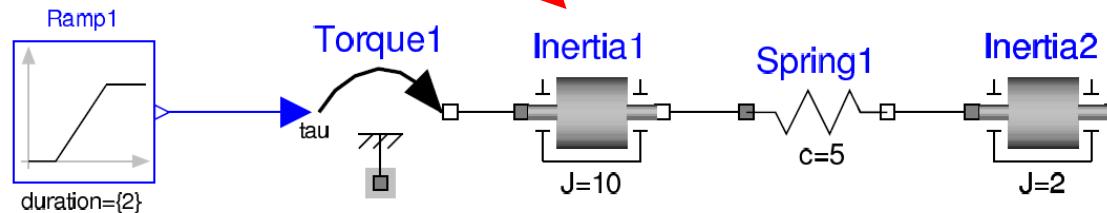


<http://www.modelica.org>

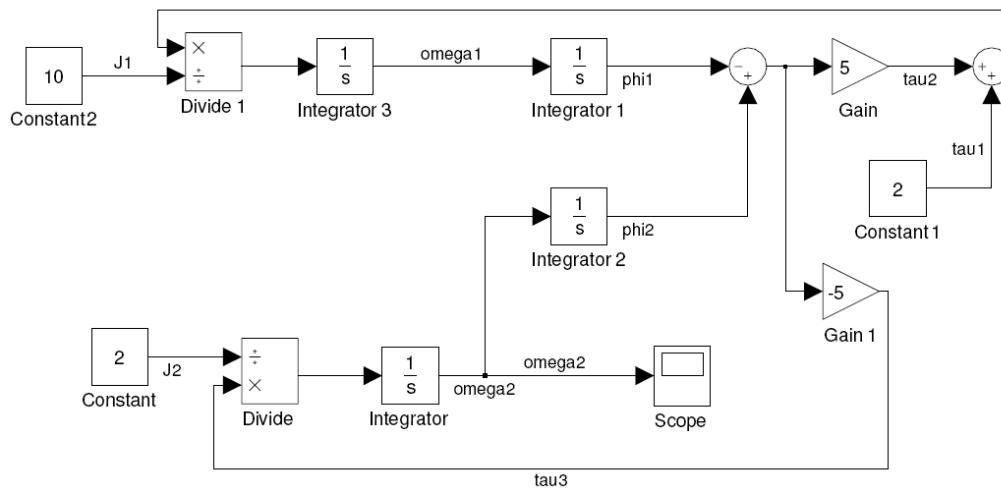


Keeps the physical structure

## Acausal model (Modelica)



## Causal block-based model (Simulink)



## Equation-based (computationally a-causal) modelling

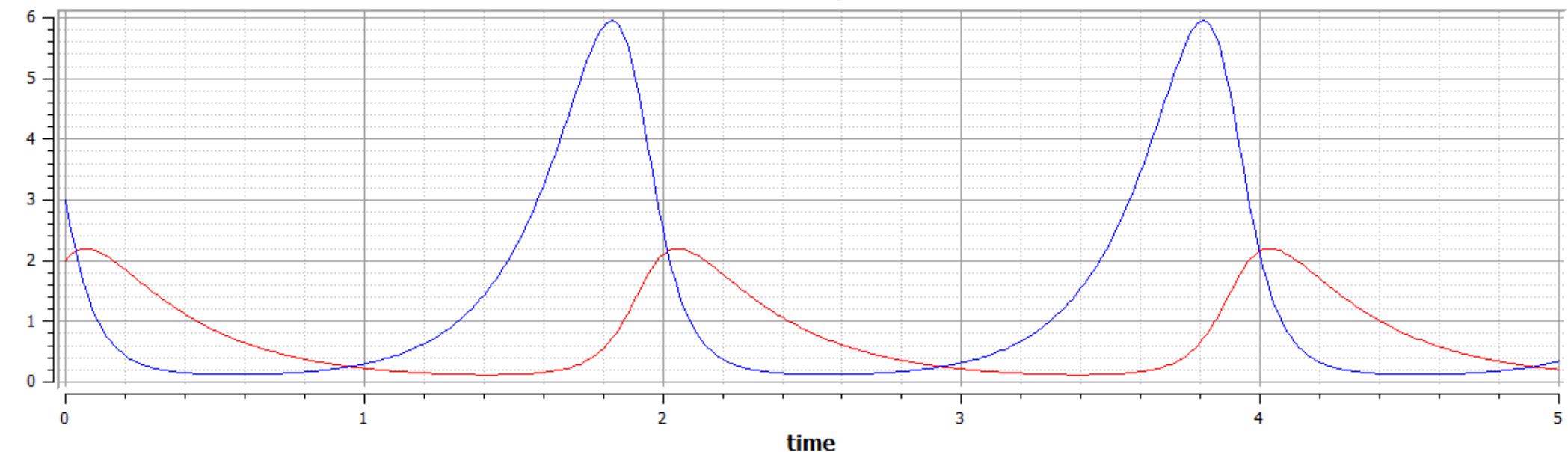
```
model mySimpleEqnSet "simple equation set"
    Real x(start=2, fixed=true);
    Real y(start=3, fixed=true);
equation
    der(x) = 2*x*y - 3*x;
    der(y) = 5*y - 7*x*y;
end mySimpleEqnSet;
```

```
plot({x,y})
```

[done]

Zoom Pan Fit in View Save Print Grid Log X Log Y

x y

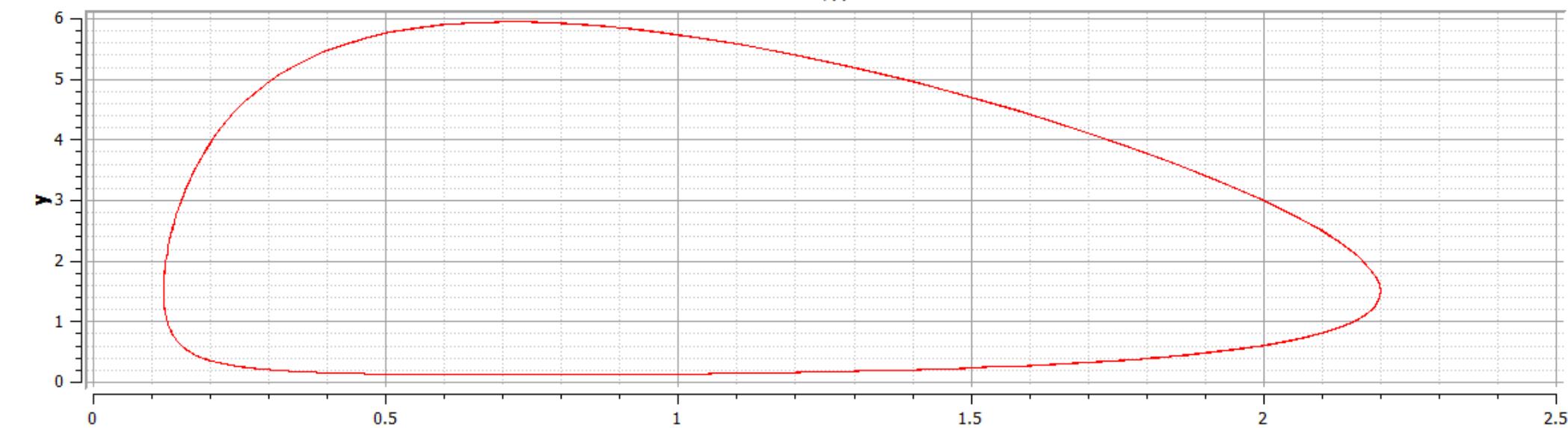


```
plotParametric(x,y)
```

[done]

Zoom Pan Fit in View Save Print Grid Log X Log Y

y(x)

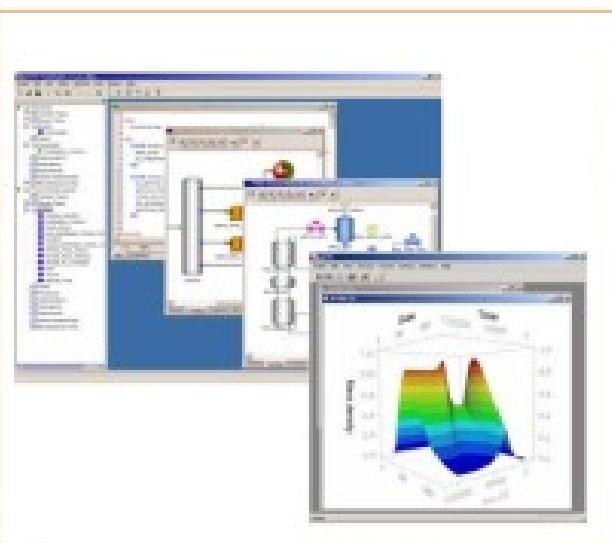


- Model exchange/re-use standard (Modelica Association)
- Modelica Standard Library (MSL)
- Object-oriented, hierarchical; semantics based on flattening
- Computationally a-causal modelling; semantics based on DAEs
- Originated in Hilding Elmquist's 1978 PhD thesis @ Lund
- Early 1990's: Modelica Design Team  
(started in SiE – Simulation in Europe ESPRIT Basic Research Working Group 8467)

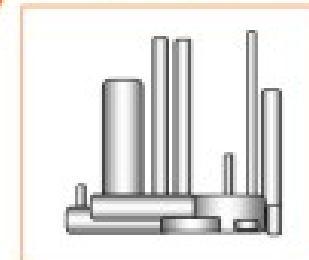
- hybrid (discrete-time/discrete-event) constructs (e.g., used to model network protocols based on TrueTime <http://www.control.lth.se/truetime/>)
- Limited support for Dynamic Structure models (i.e., no “agents”)
- Separate model from its (numerical) solution ...
- Generate Functional Mockup Interface (FMI) compliant simulation units
- Currently: many commercial and open (e.g., OpenModelica) tools
- Related: Mathworks Simscape, EcosimPro, NMF, gProms, ...

## **gPROMS** **ModelBuilder**

Model development validation  
& maintenance

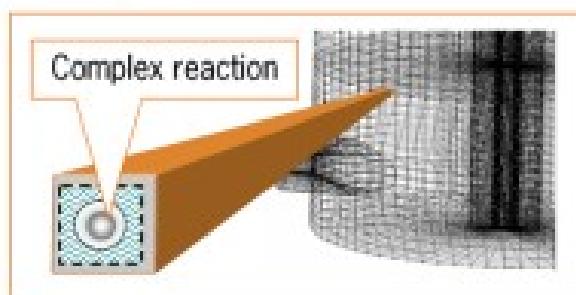


**g0:RUN**



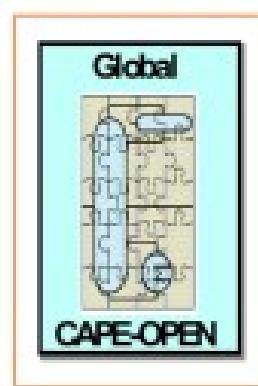
Packaged models for execution-only  
("runtime") applications

**g0:CFD**



Advanced reaction modelling for CFD tools

**g0:CAPE-OPEN**

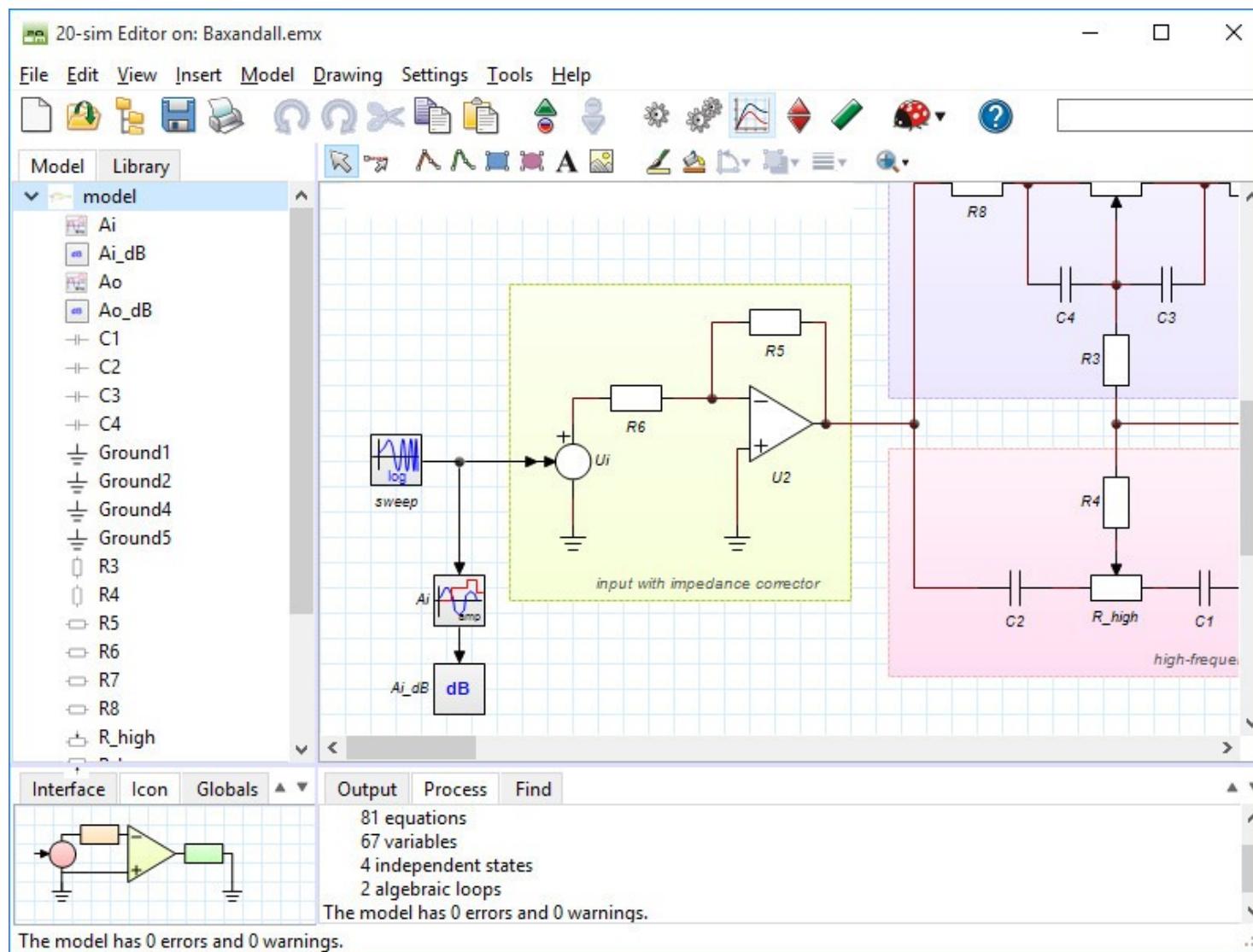


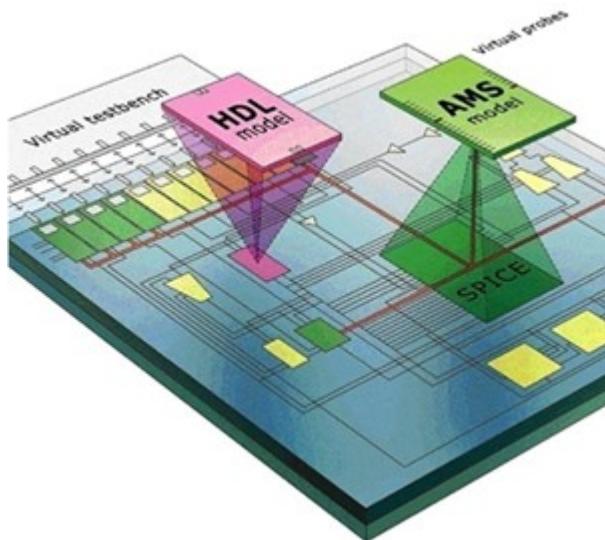
Detailed unit operation models in  
CAPE-OPEN flow-sheeting packages

**g0:Simulink**  
**g0:MATLAB**

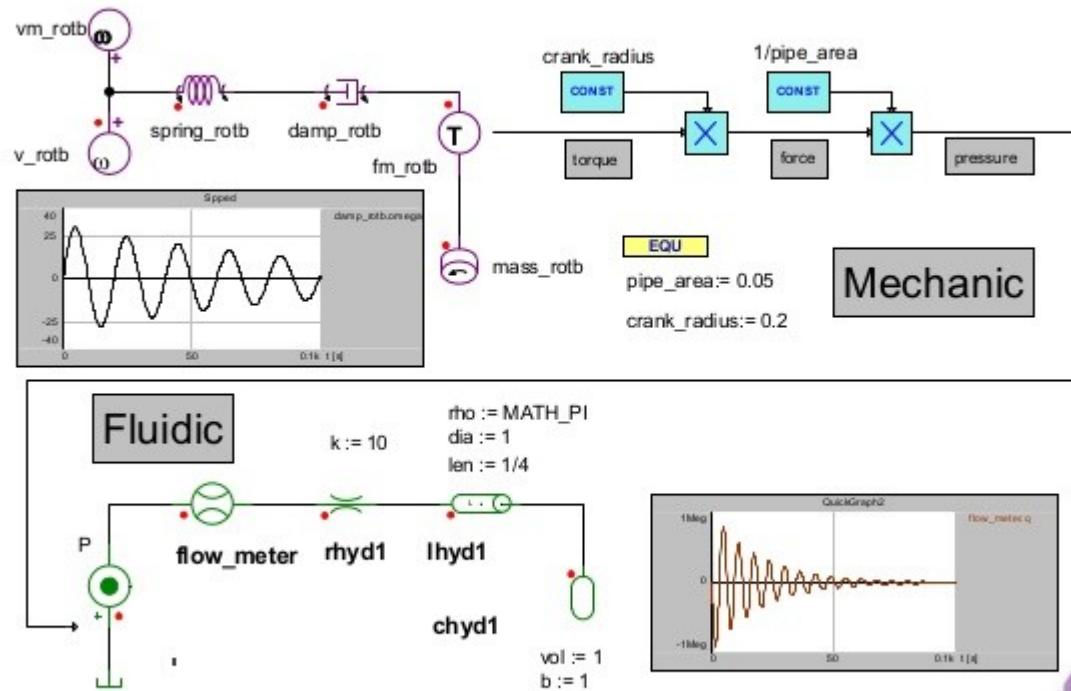


Detailed dynamic process models in  
MATLAB and Simulink®





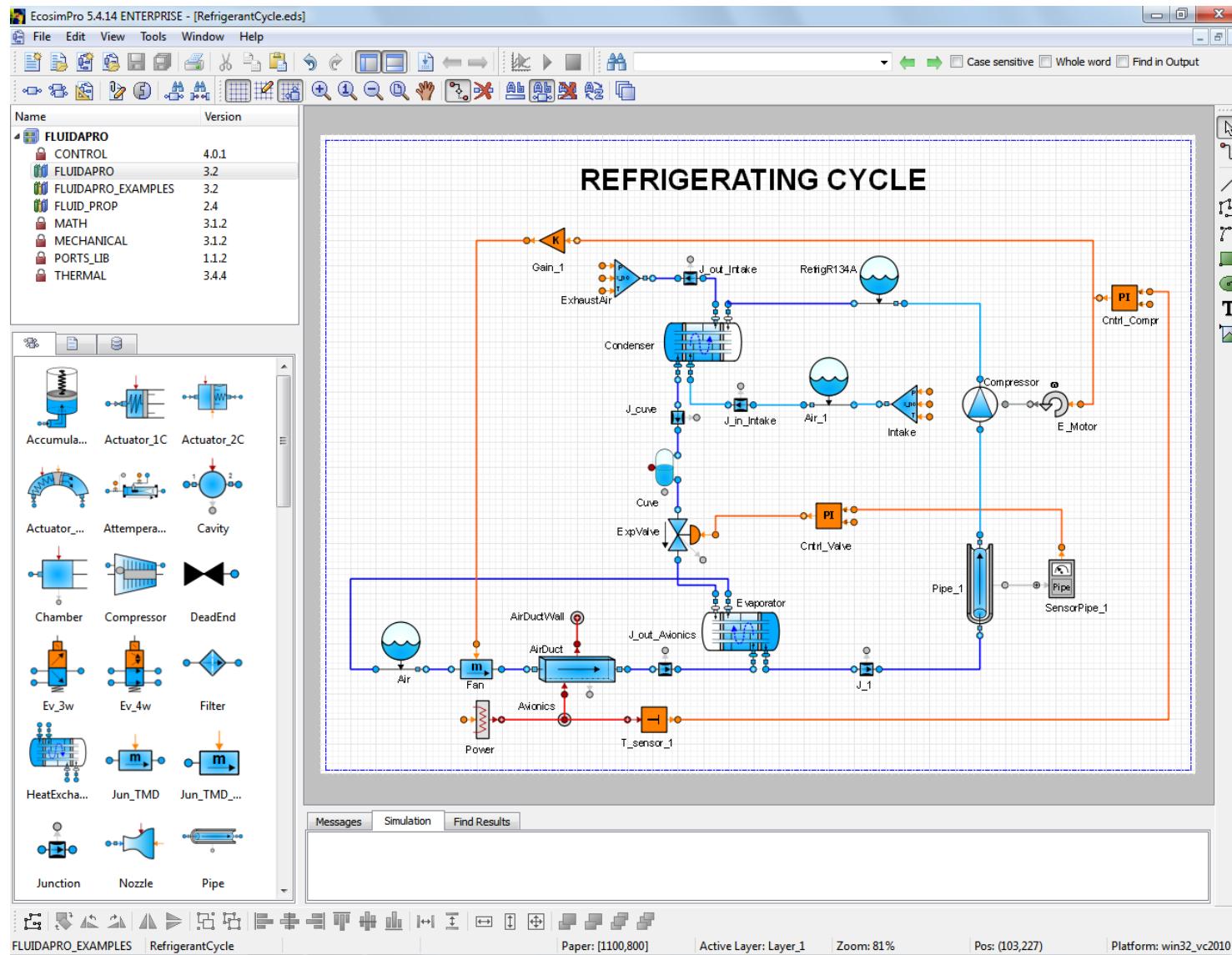
## VHDL-AMS Multi Domain Design



ANSOFT CORPORATION

# EcosimPro

## Modelling and Simulation Software



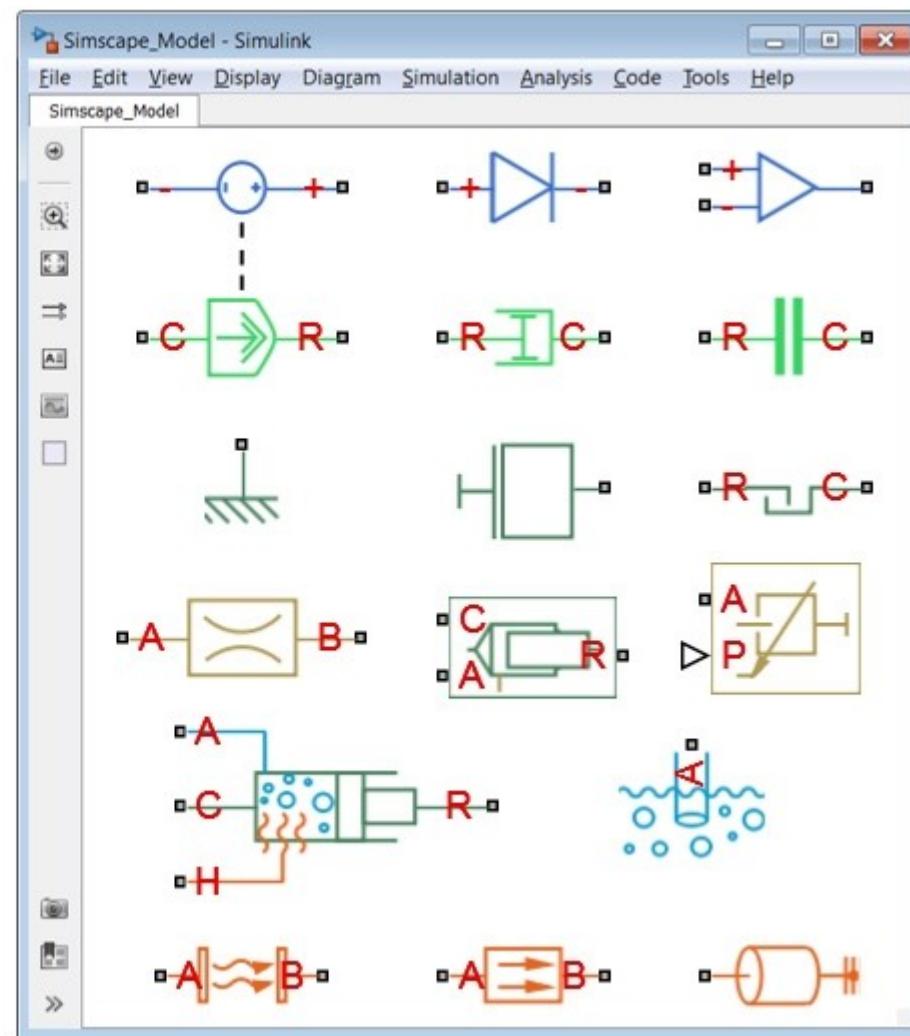


Simulink Library Browser

Enter search te...

## Simscape/Foundation Library

- ▶ Simscape
- ▶ Foundation Library
  - ◀ Electrical
    - Electrical Elements
    - Electrical Sensors
    - Electrical Sources
  - ▶ Hydraulic
  - ▶ Magnetic
  - ▶ Mechanical
  - ▶ Physical Signals
  - ▶ Pneumatic
  - ▶ Thermal
  - ▶ Thermal Liquid
  - ▶ Two-Phase Fluid



Steven Xu

## Electrical Types

```
type Time = Real (final quantity="Time", final unit="s");
type ElectricPotential = Real (final quantity="ElectricPotential",
                                final unit="V");
type Voltage = ElectricPotential;
type ElectricCurrent = Real (final quantity="ElectricCurrent",
                             final unit="A");
type Current = ElectricCurrent;
```

Beware: variables are **signals** (functions of **time**)!

Libraries

- VolumeDensityOfCharge
- SurfaceDensityOfCharge
- ElectricFieldStrength
- ElectricPotential**

Writable Type Modelica Text View C:/OpenModelica1.9.1Beta2/lib/omlibrary/Modelica 3.2.1/SIunits.mo Line: 1, Col: 0

```
1 type ElectricPotential = Real(final quantity = "ElectricPotential", final unit = "V");
```

Libraries

- VolumeDensityOfCharge
- SurfaceDensityOfCharge
- ElectricFieldStrength
- ElectricPotential**
- Voltage

Writable Type Modelica Text View C:/Open...nts.mo Line: 1, Col: 0

```
1 type Voltage = ElectricPotential;
```

## Electrical Pin Interface

```
connector PositivePin "Positive pin of an electric component"
    Voltage v "Potential at the pin";
    flow Current i "Current flowing into the pin";
end PositivePin;
```

Libraries

-  CCC
-  OpAmp
-  OpAmpDetailed
-  VariableResistor
-  VariableConductor
-  VariableCapacitor
-  VariableInductor
-  Ideal
-  Interfaces
  -  Pin
  -  PositivePin
  -  NegativePin
  -  TwoPin
  -  OnePort
  -  TwoPort
  -  ConditionalHeatPort
  -  AbsoluteSensor
  -  RelativeSensor
  -  VoltageSource
  -  CurrentSource
-  Lines
-  Semiconductors
-  Sensors
-  Sources
-  Digital
-  Machines

File Writeable Connector Modelica Text View C:/OpenModelica1.9.1Beta2/lib/omlibrary/Modelica 3.2.1/Electrical/Analog/Interfaces.mo Line: 1, Col: 0

```

1 connector PositivePin "Positive pin of an electric component"
2   Modelica.SIunits.Voltage v "Potential at the pin" annotation(unassignedMessage = "An electrical
3   potential cannot be uniquely calculated.
4   The reason could be that
5   - a ground object is missing (Modelica.Electrical.Analog.Basic.Ground)
6   to define the zero potential of the electrical circuit, or
7   - a connector of an electrical component is not connected.");
8   flow Modelica.SIunits.Current i "Current flowing into the pin" annotation(unassignedMessage = "An
9   electrical current cannot be uniquely calculated.
10  The reason could be that
11  - a ground object is missing (Modelica.Electrical.Analog.Basic.Ground)
12  to define the zero potential of the electrical circuit, or
13  - a connector of an electrical component is not connected.");
14  annotation(defaultComponentName = "pin_p", Documentation(info = "<html>
15 <p>Connectors PositivePin and NegativePin are nearly identical. The only difference is that the
16 icons are different in order to identify more easily the pins of a component. Usually, connector
17 PositivePin is used for the positive and connector NegativePin for the negative pin of an electrical
18 component.</p>
19 </html>"), revisions = "<html>
20 <ul>
21 <li><i> 1998 </i>
22   by Christoph Clauss<br> initially implemented<br>
23 </li>
24 </ul>
25 </html>"), Icon(coordinateSystem(preserveAspectRatio = true, extent = {{-100,-100},{100,100}}),
26 graphics = {Rectangle(extent = {{-100,100},{100,-100}}, lineColor = {0,0,255}, fillColor =
27 {0,0,255}, fillPattern = FillPattern.Solid)}), Diagram(coordinateSystem(preserveAspectRatio = true,
28 extent = {{-100,-100},{100,100}}), graphics = {Rectangle(extent = {{-40,40},{40,-40}}, lineColor =
29 {0,0,255}, fillColor = {0,0,255}, fillPattern = FillPattern.Solid),Text(extent = {{-160,110},
30 {40,50}}, lineColor = {0,0,255}, textString = "%name")}));
31 end PositivePin;

```

## Electrical Port

```
partial model OnePort
  "Component with two electrical pins p and n
   and current i from p to n"
  Voltage v "Voltage drop between the two pins (= p.v - n.v)";
  Current i "Current flowing from pin p to pin n";
  PositivePin p;
  NegativePin n;
equation
  v = p.v - n.v;
  0 = p.i + n.i;
  i = p.i;
end OnePort;
```

Libraries

- CCC
- OpAmp
- + OpAmpDetailed
- VariableResistor
- VariableConductor
- VariableCapacitor
- VariableInductor
- + Ideal
- Interfaces
- Pin
- PositivePin
- NegativePin
- TwoPin
- OnePort
- TwoPort
- ConditionalHeatPort
- AbsoluteSensor
- RelativeSensor
- VoltageSource
- CurrentSource
- + Lines
- + Semiconductors
- + Sensors
- + Sources
- + Digital
- + Machines
- + MultiPhase

File: C:/OpenModelica1.9.1Beta2/lib/omlibrary/Modelica 3.2.1/Electrical/Analog/Interfaces.mo | Line: 1, Col: 0

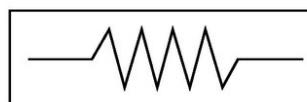
```

1 partial model OnePort "Component with two electrical pins p and n and current i from p to n"
2   SI.Voltage v "Voltage drop between the two pins (= p.v - n.v)";
3   SI.Current i "Current flowing from pin p to pin n";
4   PositivePin p "Positive pin (potential p.v > n.v for positive voltage drop v)"
5     annotation(Placement(transformation(extent = {{-110,-10},{-90,10}}), rotation = 0));
6   NegativePin n "Negative pin" annotation(Placement(transformation(extent = {{110,-10},{90,10}}),
7     rotation = 0));
8   equation
9     v = p.v - n.v;
10    0 = p.i + n.i;
11    i = p.i;
12    annotation(Documentation(info = "<html>
13      <p>Superclass of elements which have <b>two</b> electrical pins: the positive pin connector
14      <i>p</i>, and the negative pin connector <i>n</i>. It is assumed that the current flowing into pin p
15      is identical to the current flowing out of pin n. This current is provided explicitly as current
16      i.</p>
17    </html>"), revisions = "<html>
18      <ul>
19        <li><i> 1998 </i>
20          by Christoph Clauss<br> initially implemented<br>
21        </li>
22      </ul>
23    </html>"), Diagram(coordinateSystem(preserveAspectRatio = true, extent = {{-100,-100},{100,100}}),
24      graphics = {Line(points = {{-110,20}, {-85,20}}, color = {160,160,164}), Polygon(points = {{-95,23},
25        {-85,20}, {-95,17}, {-95,23}}, lineColor = {160,160,164}, fillColor = {160,160,164}, fillPattern =
26        FillPattern.Solid), Line(points = {{90,20}, {115,20}}, color = {160,160,164}), Line(points = {{-125,0},
27        {-115,0}}, color = {160,160,164}), Line(points = {{-120,-5}, {-120,5}}, color =
28        {160,160,164}), Text(extent = {{-110,25}, {-90,45}}, lineColor = {160,160,164}, textString =
29        "i"), Polygon(points = {{105,23}, {115,20}, {105,17}, {105,23}}, lineColor = {160,160,164}, fillColor =
30        {160,160,164}, fillPattern = FillPattern.Solid), Line(points = {{115,0}, {125,0}}, color =
31        {160,160,164}), Text(extent = {{90,45}, {110,25}}, lineColor = {160,160,164}, textString = "i"))});
32 end OnePort;

```

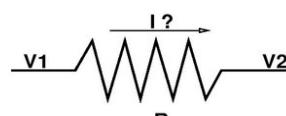
## Object-oriented re-use and causality

## Electrical Resistor

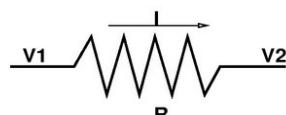


$$V_1 - V_2 = R \cdot I$$

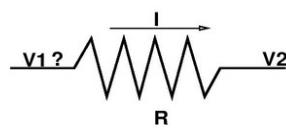
Object "resistor"



$$I = (V_1 - V_2)/R$$



$$V_2 = V_1 - R \cdot I$$



$$V_1 = V_2 + R \cdot I$$

```
model Resistor "Ideal linear electrical resistor"
  extends OnePort;
  parameter Resistance R=1 "Resistance";
  equation
    R*i = v;
end Resistor;
```

OMEdit - OpenModelica Connection Editor

File Edit View Simulation FMI XML Tools Help

Libraries Browser

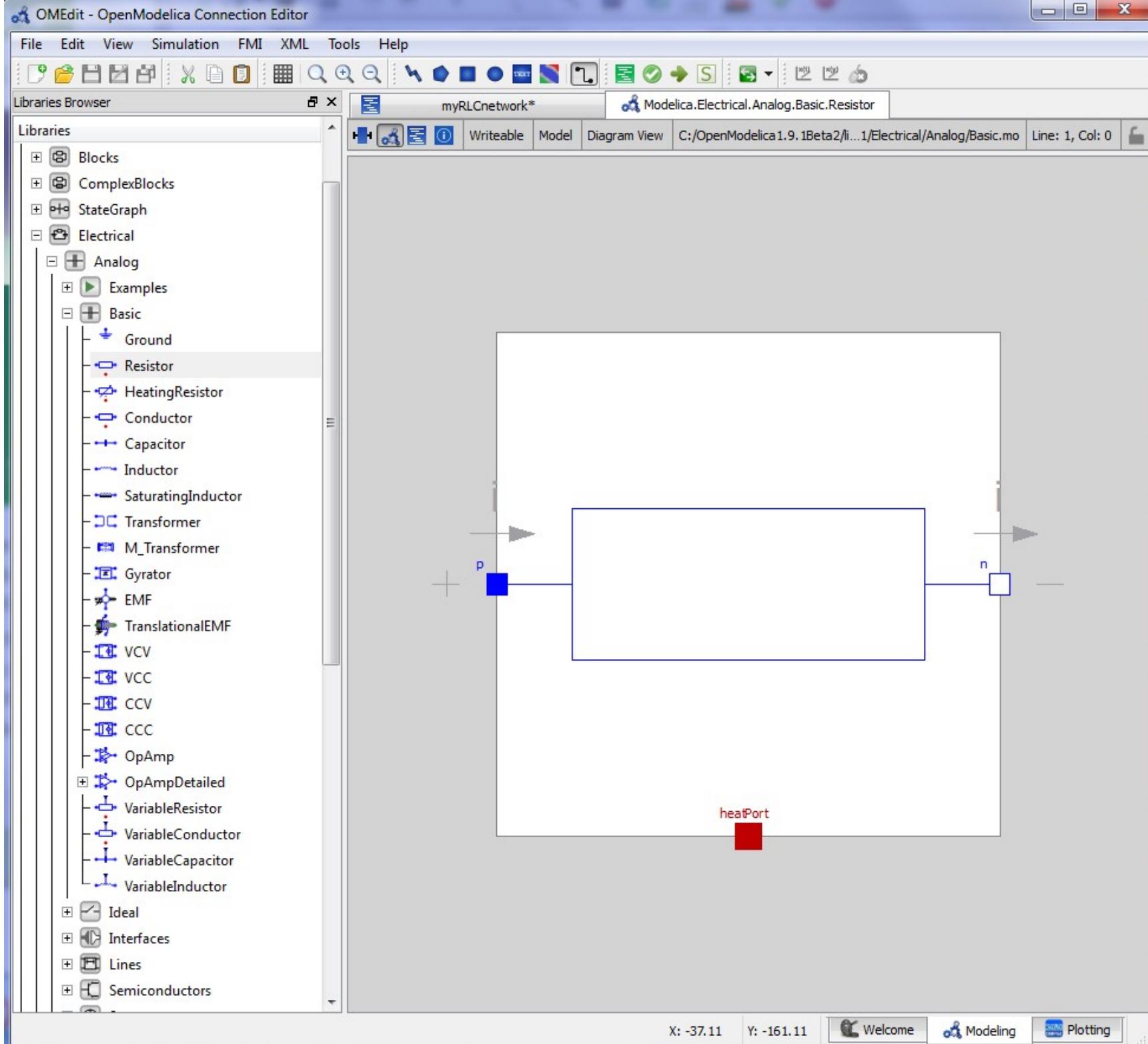
myRLCnetwork\* Modelica.Electrical.Analog.Basic.Resistor

C:/OpenModelica1.9.1Beta2/lib/omlibrary/Modelica 3.2.1/Electrical/Analog/Basic.mo Line: 1, Col: 0

```

1 model Resistor "Ideal linear electrical resistor"
2   parameter Modelica.SIunits.Resistance R(start = 1) "Resistance at temperature T_ref";
3   parameter Modelica.SIunits.Temperature T_ref = 300.15 "Reference temperature";
4   parameter Modelica.SIunits.LinearTemperatureCoefficient alpha = 0 "Temperature coefficient of resistance
(R_actual = R*(1 + alpha*(T_heatPort - T_ref)));
5   extends Modelica.Electrical.Analog.Interfaces.OnePort;
6   extends Modelica.Electrical.Analog.Interfaces.ConditionalHeatPort(T = T_ref);
7   Modelica.SIunits.Resistance R_actual "Actual resistance = R*(1 + alpha*(T_heatPort - T_ref))";
8 equation
9   assert(1 + alpha * (T_heatPort - T_ref) >= Modelica.Constants.eps, "Temperature outside scope of model!");
10  R_actual = R * (1 + alpha * (T_heatPort - T_ref));
11  v = R_actual * i;
12  LossPower = v * i;
13  annotation(Documentation(info = "<html>
14 <p>The linear resistor connects the branch voltage <i>v</i> with the branch current <i>i</i> by <i>i*R = v</i>.
The Resistance <i>R</i> is allowed to be positive, zero, or negative.</p>
15 </html>"), revisions = "<html>
16 <ul>
17 <li><i> August 07, 2009 </i>
18     by Anton Haumer<br> temperature dependency of resistance added<br>
19   </li>
20 <li><i> March 11, 2009 </i>
21     by Christoph Clauss<br> conditional heat port added<br>
22   </li>
23 <li><i> 1998 </i>
24     by Christoph Clauss<br> initially implemented<br>
25   </li>
26 </ul>
27 </html>"), Icon(coordinateSystem(preserveAspectRatio = true, extent = {{-100,-100},{100,100}}), graphics =
{Rectangle(extent = {{-70,30},{70,-30}}, lineColor = {0,0,255}, fillColor = {255,255,255}, fillPattern =
FillPattern.Solid), Line(points = {{-90,0},{-70,0}}, color = {0,0,255}), Line(points = {{70,0},{90,0}}, color =
{0,0,255}), Text(extent = {{-144,-40},{142,-72}}, lineColor = {0,0,0}, textString = "R=%R"), Line(visible =
useHeatPort, points = {{0,-100},{0,-30}}, color = {127,0,0}, smooth = Smooth.None, pattern =
LinePattern.Dot), Text(extent = {{-152,87},{148,47}}, textString = "%name", lineColor = {0,0,255})), Diagram(coordinateSystem(preserveAspectRatio = true, extent = {{-100,-100},{100,100}}), graphics =
{Rectangle(extent = {{-70,30},{70,-30}}, lineColor = {0,0,255}), Line(points = {{-96,0},{-70,0}}, color =
{0,0,255}), Line(points = {{70,0},{96,0}}, color = {0,0,255}))});
28 end Resistor;
```

X: -15.03 Y: 154.06 Welcome Modeling Plotting

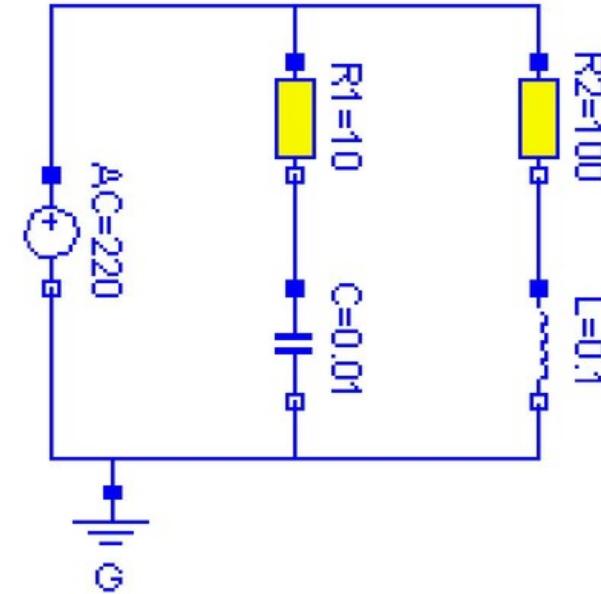


## The circuit

```

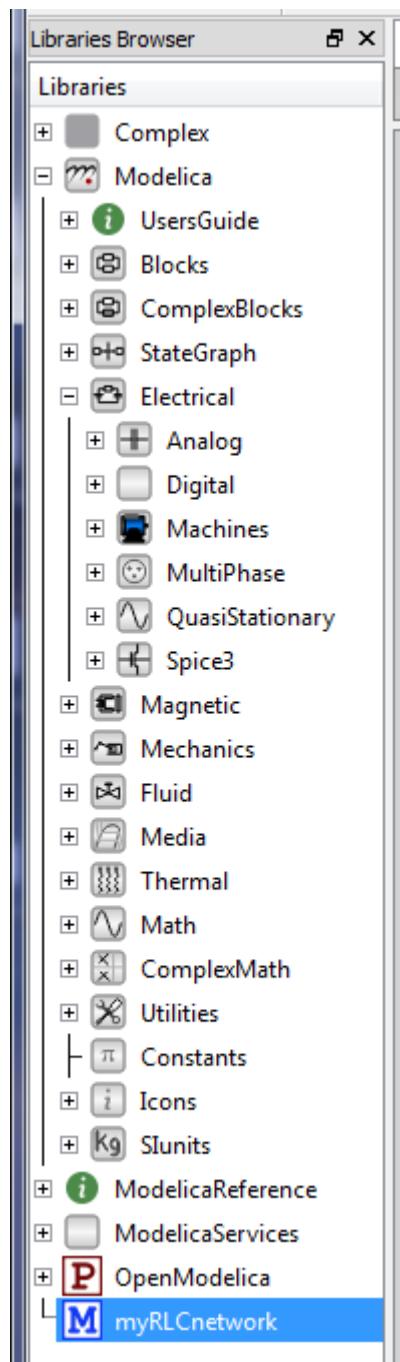
model circuit
  Resistor R1(R=10);
  Capacitor C(C=0.01);
  Resistor R2(R=100);
  Inductor L(L=0.1);
  VsourceAC AC;
  Ground G;
equation
  connect(AC.p, R1.p);
  connect(R1.n, C.p);
  connect(C.n, AC.n);
  connect(R1.p, R2.p);
  connect(R2.n, L.p);
  connect(L.n, C.n);
  connect(AC.n, G.p);
end circuit;

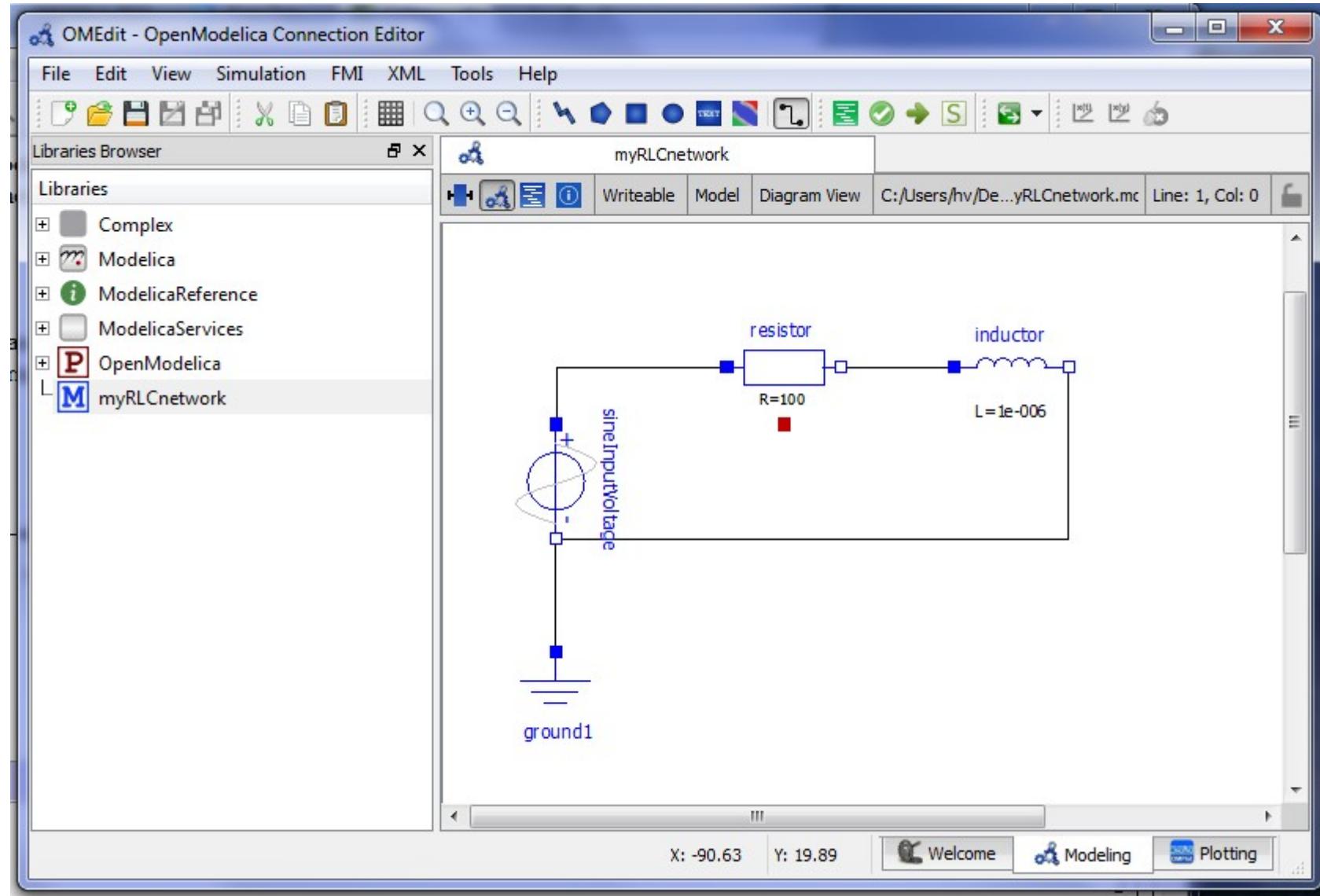
```



Meaning: set of Differential Algebraic Equations (DAEs) obtained by

- 1.a. expanding inheritance
- 1.b. instantiation
2. flattening hierarchy, construct unique names
3. expanding connect() into equations (across vs. flow)





OMEdit - OpenModelica Connection Editor

File Edit View Simulation FMI XML Tools Help

Libraries Browser

Libraries

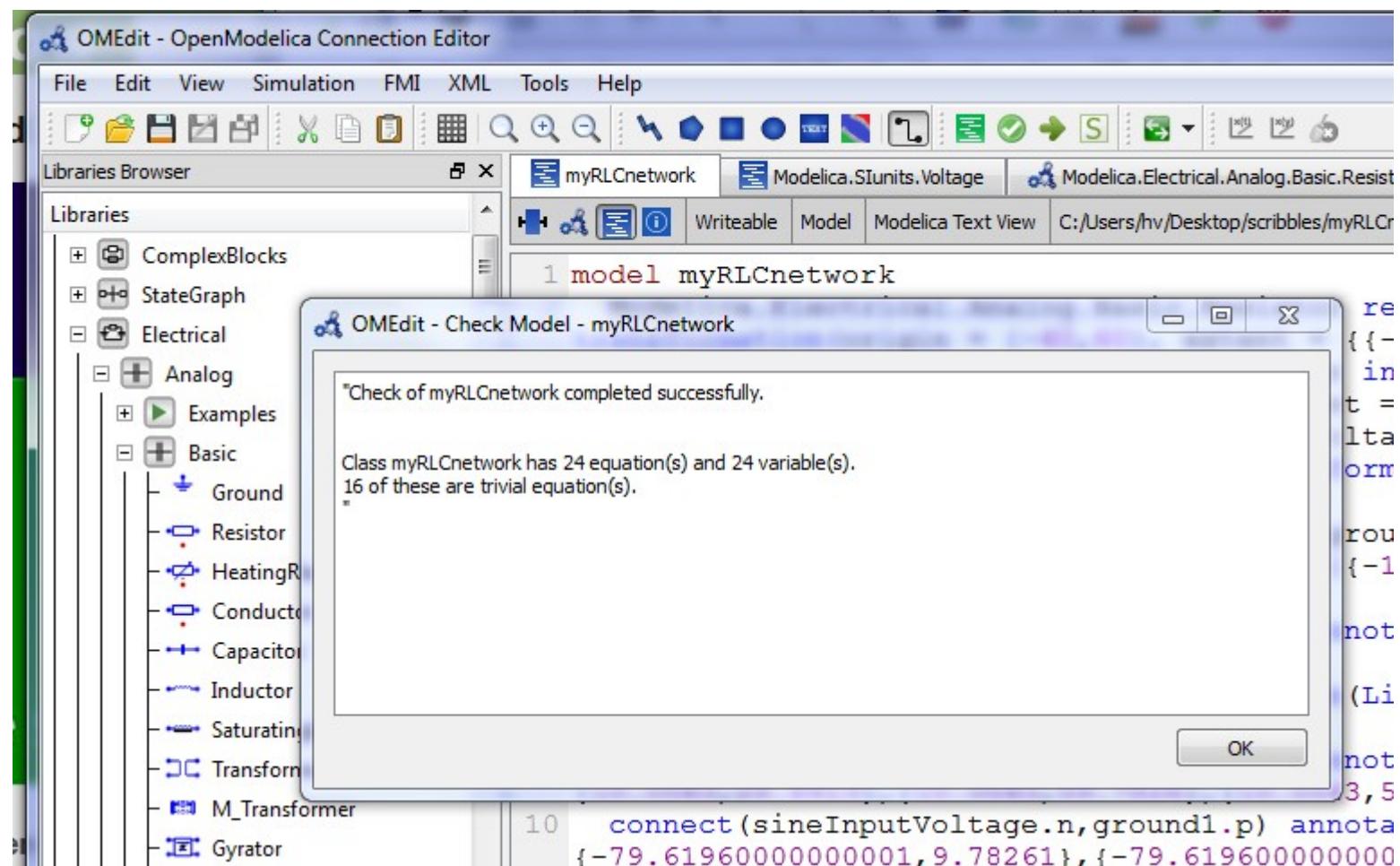
- + Complex
- + Modelica
- + ModelicaReference
- + ModelicaServices
- + OpenModelica
- L myRLCnetwork**

myRLCnetwork

Writeable Model Modelica Text View C:/Users/hv/Desktop/scribbles/myRLCnetwork.mo Line: 1, Col: 11

```
1 model myRLCnetwork
2   Modelica.Electrical.Analog.Basic.Resistor resistor(R = 100) annotation(Placement(visible
= true, transformation(origin = {-40,60}, extent = {{-10,-10},{10,10}}, rotation = 0)));
3   Modelica.Electrical.Analog.Inductor inductor(L = 1e-006)
annotation(Placement(visible = true, transformation(origin = {0,60}, extent = {{-10,-10},
{10,10}}, rotation = 0));
4   Modelica.Electrical.Analog.Sources.SineVoltage sineInputVoltage(V = 10, freqHz = 50)
annotation(Placement(visible = true, transformation(origin = {-80,40}, extent =
{{-10,-10},{10,10}}, rotation = -90)));
5   Modelica.Electrical.Analog.Basic.Ground ground1 annotation(Placement(visible = true,
transformation(origin = {-80,0}, extent = {{-10,-10},{10,10}}, rotation = 0)));
6 equation
7   connect(resistor.p,sineInputVoltage.p) annotation(Line(points = {{-50,60}, {-79.8913,60},
{-79.8913,49.4565}, {-79.8913,49.4565}}));
8   connect(resistor.n,inductor.p) annotation(Line(points = {{-30,60}, {-9.78261,60},
{-9.78261,59.2391}, {-9.78261,59.2391}}));
9   connect(sineInputVoltage.n,inductor.n) annotation(Line(points = {{-80,30}, {-80,29.8913},
{10.0543,29.8913}, {10.0543,59.7826}, {10.0543,59.7826}}));
10  connect(sineInputVoltage.n,ground1.p) annotation(Line(points = {{-80,30}, {-80,9.78261},
{-79.61960000000001,9.78261}, {-79.61960000000001,9.78261}}));
11  annotation(Icon(coordinateSystem(extent = {{-100,-100},{100,100}}, preserveAspectRatio =
true, initialScale = 0.1, grid = {2,2})), Diagram(coordinateSystem(extent = {{-100,-100},
{100,100}}, preserveAspectRatio = true, initialScale = 0.1, grid = {2,2})));
12 end myRLCnetwork;
```

X: -90.63 Y: 19.89 Welcome Modeling Plotting



```

class myRLCnetwork
Real resistor.v(quantity = "ElectricPotential", unit = "V") "Voltage drop between the two pins (= p.v - n.v)";
Real resistor.i(quantity = "ElectricCurrent", unit = "A") "Current flowing from pin p to pin n";
Real resistor.p.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real resistor.p.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
Real resistor.n.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real resistor.n.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
parameter Boolean resistor.useHeatPort = false "=true, if HeatPort is enabled";
Real resistor.LossPower(quantity = "Power", unit = "W") "Loss power leaving component via HeatPort";
Real resistor.T_heatPort(quantity = "ThermodynamicTemperature", unit = "K", displayUnit = "degC", min = 0.0, start = 288.15, nominal = 300.0) "Temperature of HeatPort";
parameter Real resistor.R(quantity = "Resistance", unit = "Ohm", start = 1.0) = 100.0 "Resistance at temperature T_ref";
parameter Real resistor.T_ref(quantity = "ThermodynamicTemperature", unit = "K", displayUnit = "degC", min = 0.0, start = 288.15, nominal = 300.0) = 300.15 "Reference temperature";
parameter Real resistor.alpha(quantity = "LinearTemperatureCoefficient", unit = "1/K") = 0.0 "Temperature coefficient of resistance (R_actual = R*(1 + alpha*(T_heatPort - T_ref)))";
Real resistor.R_actual(quantity = "Resistance", unit = "Ohm") "Actual resistance = R*(1 + alpha*(T_heatPort - T_ref))";
parameter Real resistor.T(quantity = "ThermodynamicTemperature", unit = "K", displayUnit = "degC", min = 0.0, start = 288.15, nominal = 300.0) = resistor.T_ref "Fixed device temperature if useHeatPort = false";
Real inductor.v(quantity = "ElectricPotential", unit = "V") "Voltage drop between the two pins (= p.v - n.v)";
Real inductor.i(quantity = "ElectricCurrent", unit = "A", start = 0.0) "Current flowing from pin p to pin n";
Real inductor.p.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real inductor.p.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
Real inductor.n.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real inductor.n.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
parameter Real inductor.L(quantity = "Inductance", unit = "H", start = 1.0) = 1e-006 "Inductance";
Real sineInputVoltage.v(quantity = "ElectricPotential", unit = "V") "Voltage drop between the two pins (= p.v - n.v)";
Real sineInputVoltage.i(quantity = "ElectricCurrent", unit = "A") "Current flowing from pin p to pin n";
Real sineInputVoltage.p.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real sineInputVoltage.p.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
Real sineInputVoltage.n.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real sineInputVoltage.n.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";
parameter Real sineInputVoltage.offset(quantity = "ElectricPotential", unit = "V") = 0.0 "Voltage offset";
parameter Real sineInputVoltage.startTime(quantity = "Time", unit = "s") = 0.0 "Time offset";
parameter Real sineInputVoltage.V(quantity = "ElectricPotential", unit = "V", start = 1.0) = 10.0 "Amplitude of sine wave";
parameter Real sineInputVoltage.phase(quantity = "Angle", unit = "rad", displayUnit = "deg") = 0.0 "Phase of sine wave";
parameter Real sineInputVoltage.freqHz(quantity = "Frequency", unit = "Hz", start = 1.0) = 50.0 "Frequency of sine wave";
output Real sineInputVoltage.signalSource.y "Connector of Real output signal";
parameter Real sineInputVoltage.signalSource.amplitude = sineInputVoltage.V "Amplitude of sine wave";
parameter Real sineInputVoltage.signalSource.freqHz(quantity = "Frequency", unit = "Hz", start = 1.0) = sineInputVoltage.freqHz "Frequency of sine wave";
parameter Real sineInputVoltage.signalSource.phase(quantity = "Angle", unit = "rad", displayUnit = "deg") = sineInputVoltage.phase "Phase of sine wave";
parameter Real sineInputVoltage.signalSource.offset = sineInputVoltage.offset "Offset of output signal";
parameter Real sineInputVoltage.signalSource.startTime(quantity = "Time", unit = "s") = sineInputVoltage.startTime "Output = offset for time < startTime";
protected constant Real sineInputVoltage.signalSource.pi = 3.141592653589793;
Real ground1.p.v(quantity = "ElectricPotential", unit = "V") "Potential at the pin";
Real ground1.p.i(quantity = "ElectricCurrent", unit = "A") "Current flowing into the pin";

```

```

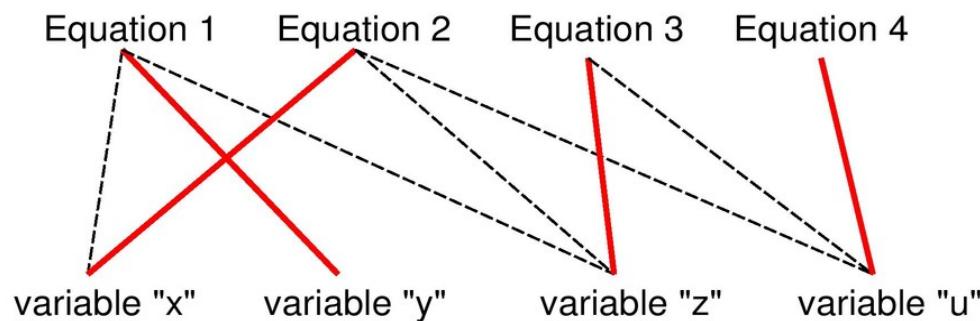
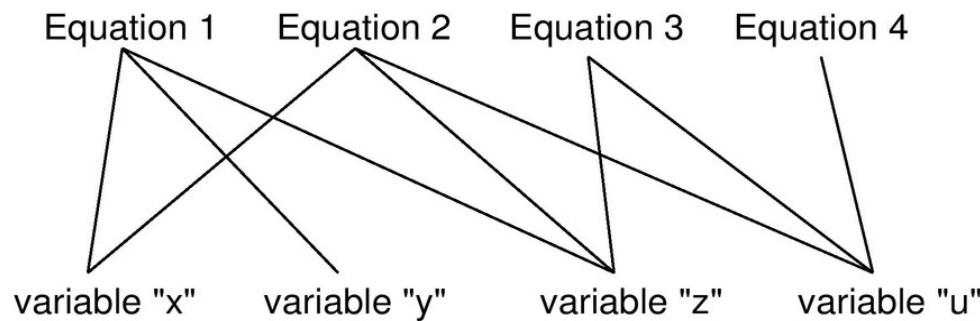
equation
  assert(1.0 + resistor.alpha * (resistor.T_heatPort - resistor.T_ref) >= 1e-015,"Temperature outside scope of model!");
  resistor.R_actual = resistor.R * (1.0 + resistor.alpha * (resistor.T_heatPort - resistor.T_ref));
  resistor.v = resistor.R_actual * resistor.i;
  resistor.LossPower = resistor.v * resistor.i;
  resistor.v = resistor.p.v - resistor.n.v;
  0.0 = resistor.p.i + resistor.n.i;
  resistor.i = resistor.p.i;
  resistor.T_heatPort = resistor.T;
  inductor.L * der(inductor.i) = inductor.v;
  inductor.v = inductor.p.v - inductor.n.v;
  0.0 = inductor.p.i + inductor.n.i;
  inductor.i = inductor.p.i;
  sineInputVoltage.signalSource.y = sineInputVoltage.signalSource.offset + (if time <
sineInputVoltage.signalSource.startTime then 0.0 else sineInputVoltage.signalSource.amplitude * sin(6.283185307179586 *
sineInputVoltage.signalSource.freqHz * (time - sineInputVoltage.signalSource.startTime) +
sineInputVoltage.signalSource.phase));
  sineInputVoltage.v = sineInputVoltage.signalSource.y;
  sineInputVoltage.v = sineInputVoltage.p.v - sineInputVoltage.n.v;
  0.0 = sineInputVoltage.p.i + sineInputVoltage.n.i;
  sineInputVoltage.i = sineInputVoltage.p.i;
  ground1.p.v = 0.0;
  resistor.p.i + sineInputVoltage.p.i = 0.0;
  resistor.n.i + inductor.p.i = 0.0;
  inductor.n.i + sineInputVoltage.n.i + ground1.p.i = 0.0;
  resistor.p.v = sineInputVoltage.p.v;
  inductor.p.v = resistor.n.v;
  ground1.p.v = inductor.n.v;
  ground1.p.v = sineInputVoltage.n.v;
end myRLCnetwork;

```

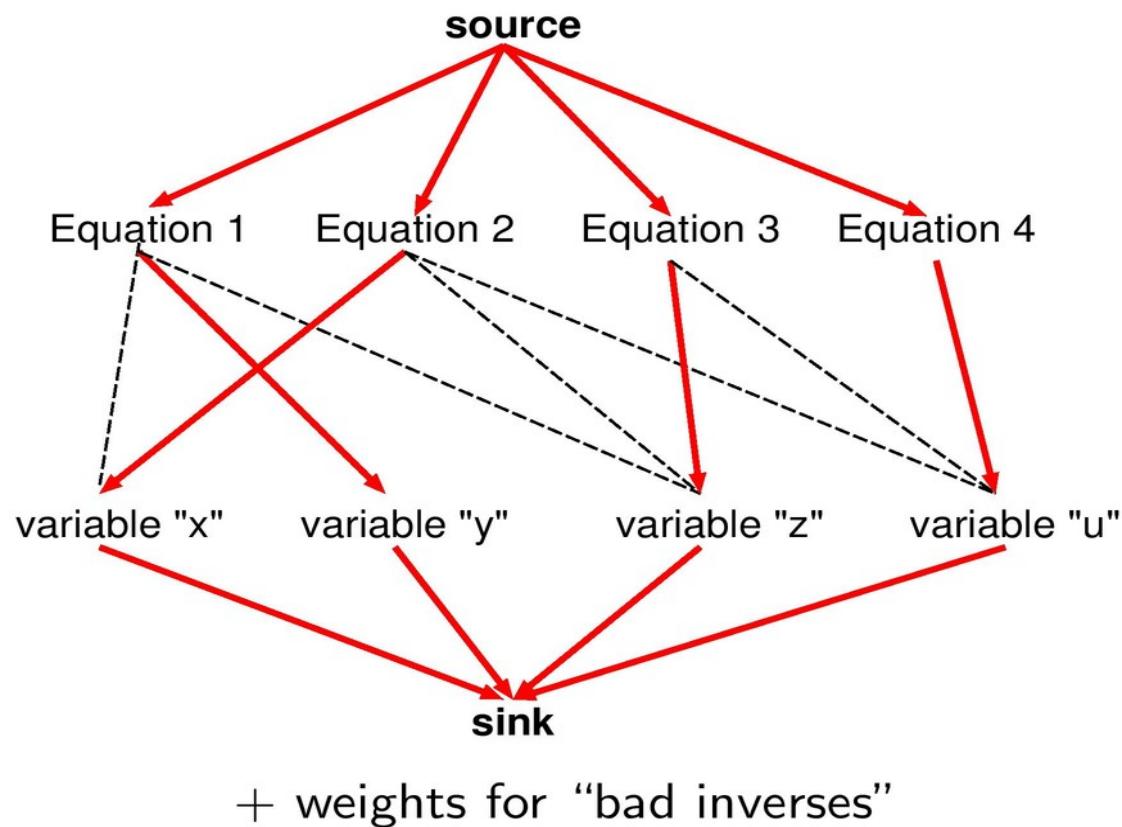
Non-causal model  
(e.g., from physical conservation laws)

$$\left\{ \begin{array}{l} x + y + z = 0 \quad \text{Equation 1} \\ x + 3z + u^2 = 0 \quad \text{Equation 2} \\ z - u - 16 = 0 \quad \text{Equation 3} \\ u - 5 = 0 \quad \text{Equation 4} \end{array} \right.$$

## Causality assignment: bipartite graph, maximum cardinality matching



## Causality assignment: network flow



## Causality assigned

$$\begin{cases} x + \underline{y} + z = 0 & \text{Equation 1} \\ \underline{x} + 3z + u^2 = 0 & \text{Equation 2} \\ \underline{z} - u - 16 = 0 & \text{Equation 3} \\ \underline{u} - 5 = 0 & \text{Equation 4} \end{cases}$$

re-write in causal form

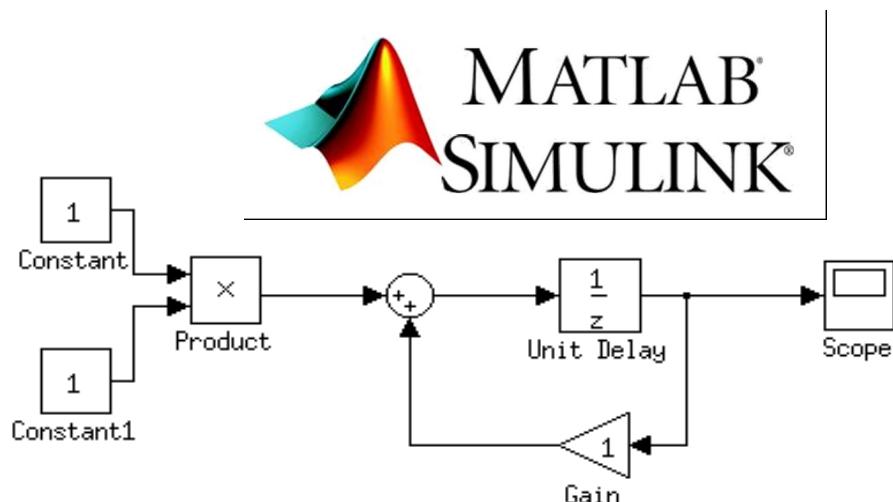
$$\begin{cases} \underline{y} = -x - z \\ \underline{x} = -3z - u^2 \\ \underline{z} = u + 16 \\ \underline{u} = 5 \end{cases}$$

# Set of Algebraic Eqns (no cyclic dependencies)

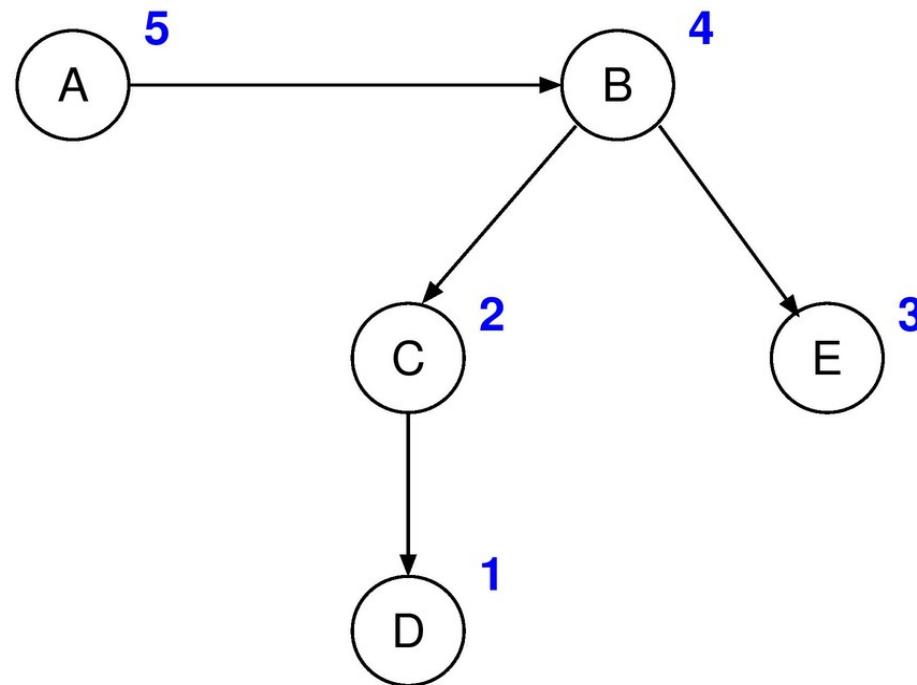
**WRONG:**

$$\begin{cases} a = b^2 + 3 \\ b = \sin(c \times e) \\ c = \sqrt{d - 4.5} \\ d = \pi/2 \\ e = u() \end{cases}$$

$$\begin{bmatrix} a &= & b^2 + 3 = 3 \\ b &= & \sin(c \times e) = 0 \\ c &= & \sqrt{d - 4.5} = \text{error} \\ d &= & \pi/2 \\ e &= & u() \end{bmatrix}$$



Sorting (no cyclic dependencies)  
DFS, postorder numbering of dependency graph



## Dependency Cycle (aka Algebraic Loop)

$$\begin{cases} x = y + 16 \\ y = -x - z \\ z = 5 \end{cases}$$

Can *never* be sorted

due to a dependency *cycle* aka *strong component*

(every vertex in the component is reachable from every other)

$$x \rightarrow y \rightarrow x$$

May be solved implicitly

$$\left[ \begin{array}{l} z = 5 \\ \left\{ \begin{array}{rcl} x - y & = & -6 \\ x + y & = & -z \end{array} \right. \end{array} \right]$$

Implicit set of  $n$  equations in  $n$  unknowns.

- non-linear  $\rightarrow$  non-linear solver.
- linear  $\rightarrow$  numerical or symbolic solution.

Linear: may be solved symbolically (Cramer)

$$x = \frac{\begin{vmatrix} -6 & -1 \\ -z & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{-6 - z}{2}; \quad y = \frac{\begin{vmatrix} 1 & -6 \\ 1 & -z \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}} = \frac{6 - z}{2}$$

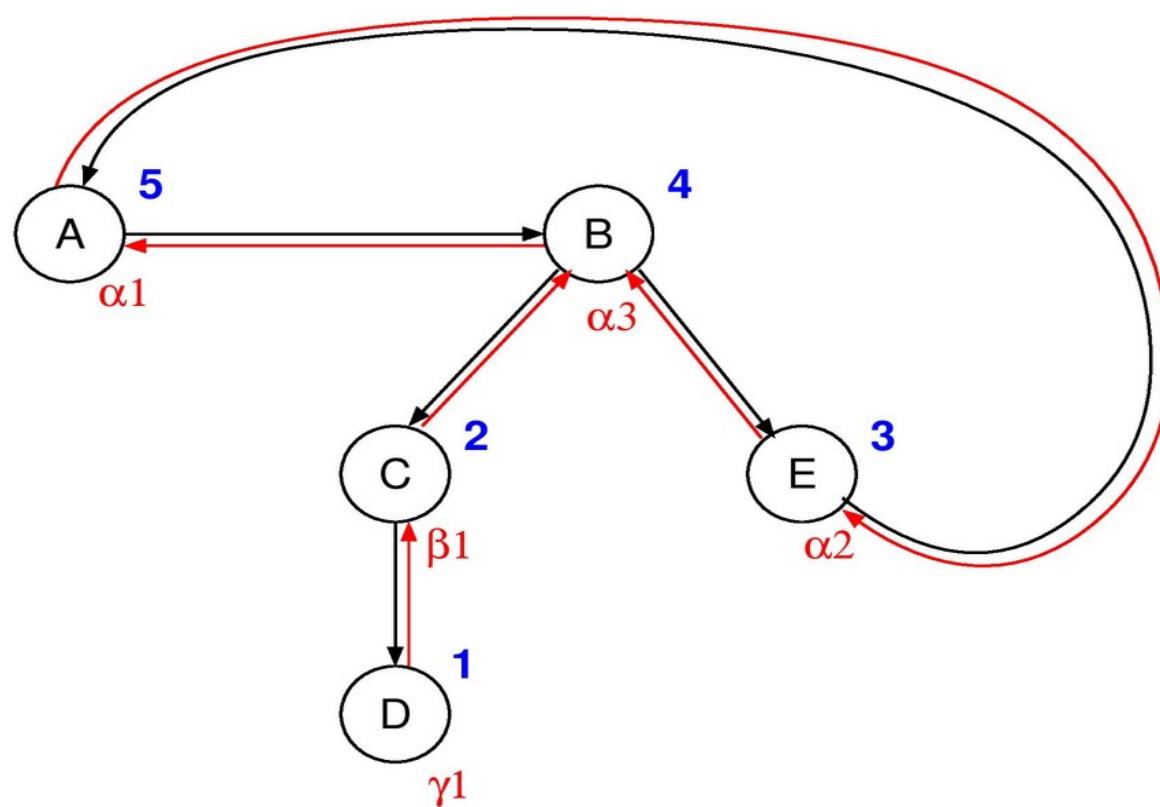
$$\begin{bmatrix} z & = & 5 \\ x & = & \frac{-6-z}{2} \\ y & = & \frac{6-z}{2} \end{bmatrix}$$

# Tarjan's algorithm for Cycle Detection

“strong components”

$$\begin{cases} a &= b^2 + 3 \\ b &= \sin(c \times e) \\ c &= \sqrt{d - 4.5} \\ d &= \pi/2 \\ e &= a^2 + u() \end{cases}$$

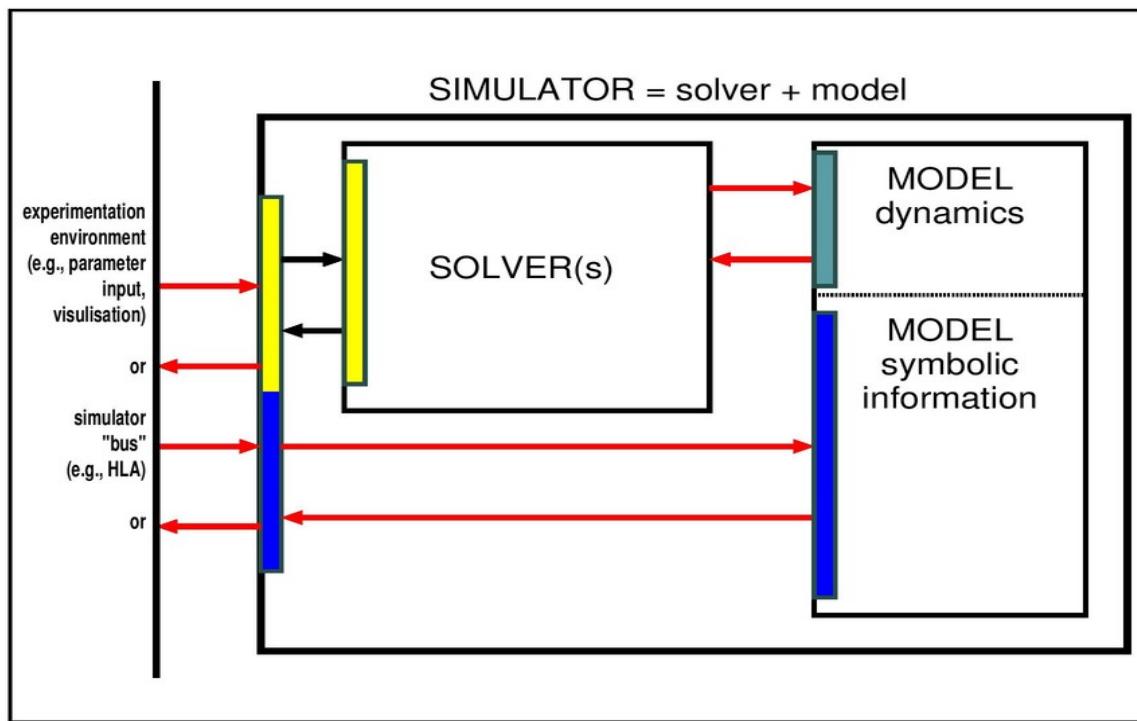
# Algebraic Loop (Cycle) Detection



## Algebraic Loop (Cycle) Detection Result

$$\left[ \begin{array}{l} d = \pi/2 \\ c = \sqrt{d - 4.5} \\ \left\{ \begin{array}{l} b = \sin(c \times e) \\ a = b^2 + 3 \\ e = a^2 + u() \end{array} \right. \end{array} \right] ; \left[ \begin{array}{l} d = \pi/2 \\ c = \sqrt{d - 4.5} \\ \left\{ \begin{array}{l} b = -\sin(c \times e) = 0 \\ a = -b^2 = -3 = 0 \\ a^2 = -e = +u() = 0 \end{array} \right. \end{array} \right]$$

# Model-Solver Interface Simulator-Environment Interface

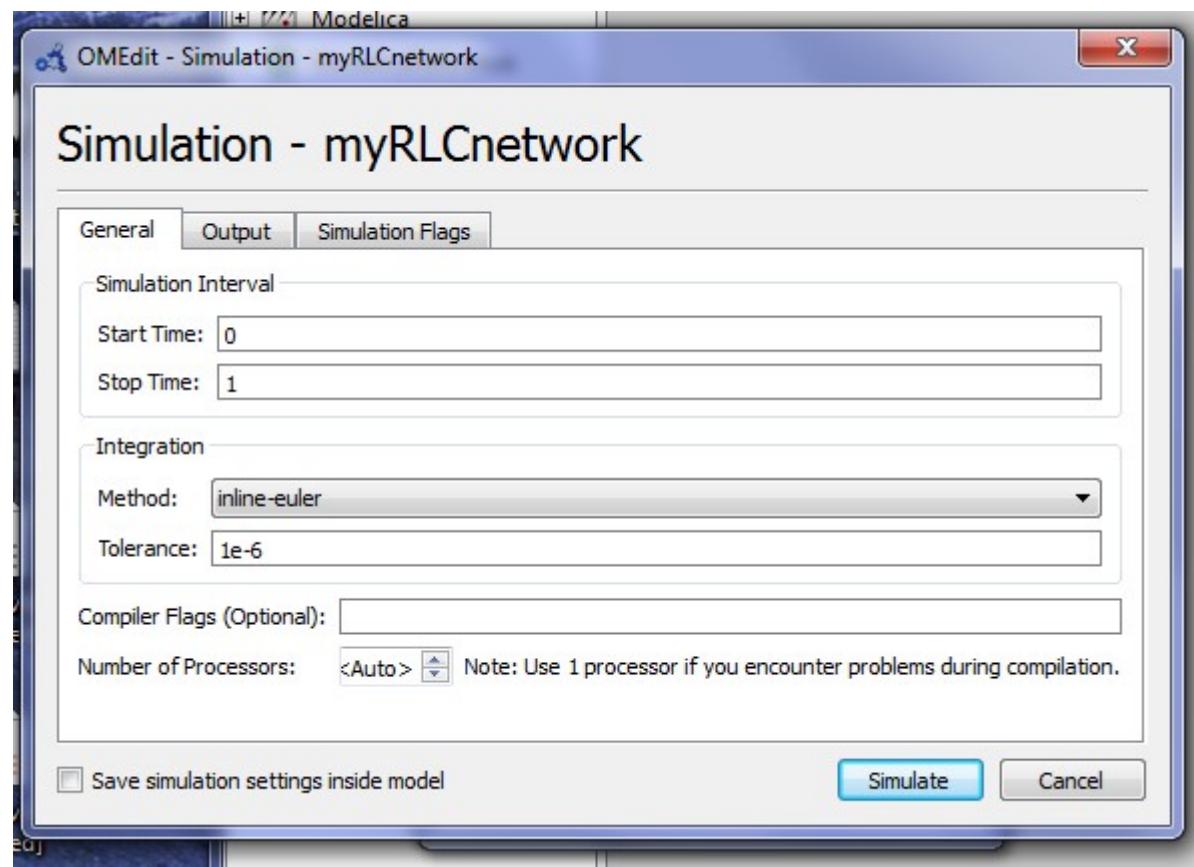


## OMEdit - myRLCnetwork Simulation Output

## Output

```
"C:\OpenModelica1.9.1Beta2\MinGW\bin\mingw32-make.exe" -j4 -f myRLCnetwork.makefile
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork.o myRLCnetwork.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_functions.o myRLCnetwork_functions.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_records.o myRLCnetwork_records.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_01exo.o myRLCnetwork_01exo.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_02nls.o myRLCnetwork_02nls.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_03lsy.o myRLCnetwork_03lsy.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_04set.o myRLCnetwork_04set.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_05evt.o myRLCnetwork_05evt.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_06inz.o myRLCnetwork_06inz.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_07dly.o myRLCnetwork_07dly.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_08bnd.o myRLCnetwork_08bnd.c
myRLCnetwork_05evt.c: In function 'myRLCnetwork_zeroCrossingDescription':
myRLCnetwork_05evt.c:51: warning: assignment discards qualifiers from pointer target type
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_09alg.o myRLCnetwork_09alg.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_10asr.o myRLCnetwork_10asr.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_11mix.o myRLCnetwork_11mix.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_12jac.o myRLCnetwork_12jac.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_13opt.o myRLCnetwork_13opt.c
gcc -falign-functions -msse2 -mfpmath=sse -I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -c -o
myRLCnetwork_14lnz.o myRLCnetwork_14lnz.c
gcc -I. -o myRLCnetwork.exe myRLCnetwork.o myRLCnetwork_functions.o myRLCnetwork_records.o myRLCnetwork_01exo.o myRLCnetwork_02nls.o
myRLCnetwork_03lsy.o myRLCnetwork_04set.o myRLCnetwork_05evt.o myRLCnetwork_06inz.o myRLCnetwork_07dly.o myRLCnetwork_08bnd.o myRLCnetwork_09alg.o
myRLCnetwork_10asr.o myRLCnetwork_11mix.o myRLCnetwork_12jac.o myRLCnetwork_13opt.o myRLCnetwork_14lnz.o -
I"C:/OpenModelica1.9.1Beta2/include/omc/c" -I. -DOPENMODELICA_XML_FROM_FILE_AT_RUNTIME -falign-functions -msse2 -mfpmath=sse -
L"C:/OpenModelica1.9.1Beta2/lib/omc" -L"C:/OpenModelica1.9.1Beta2/lib" -Wl,--stack,0x2000000,-rpath,"C:/OpenModelica1.9.1Beta2/lib/omc" -Wl,-
rpath,"C:/OpenModelica1.9.1Beta2/lib" -lregex -lexpat -lgc -lpthread -fopenmp -loleaut32 -lSimulationRuntimeC -lgc -lexpat -lregex -static-
libgcc -luuid -loleaut32 -ole32 -lws2_32 -lsundials_kinsol -lsundials_nvecserial -lipopt -lcoinmumps -lcoinmetis -lpthread -lm -lgfortranbegin -
lgfortran -lmingw32 -lgcc_eh -lmallocname -lmingwex -lmsvcrt -luser32 -lkernel32 -ladvapi32 -lshell32 -llapack-mingw -ltmglib-mingw -lblas-mingw -
lf2c -linteractive -lwsock32 -llis -lstdc++
```

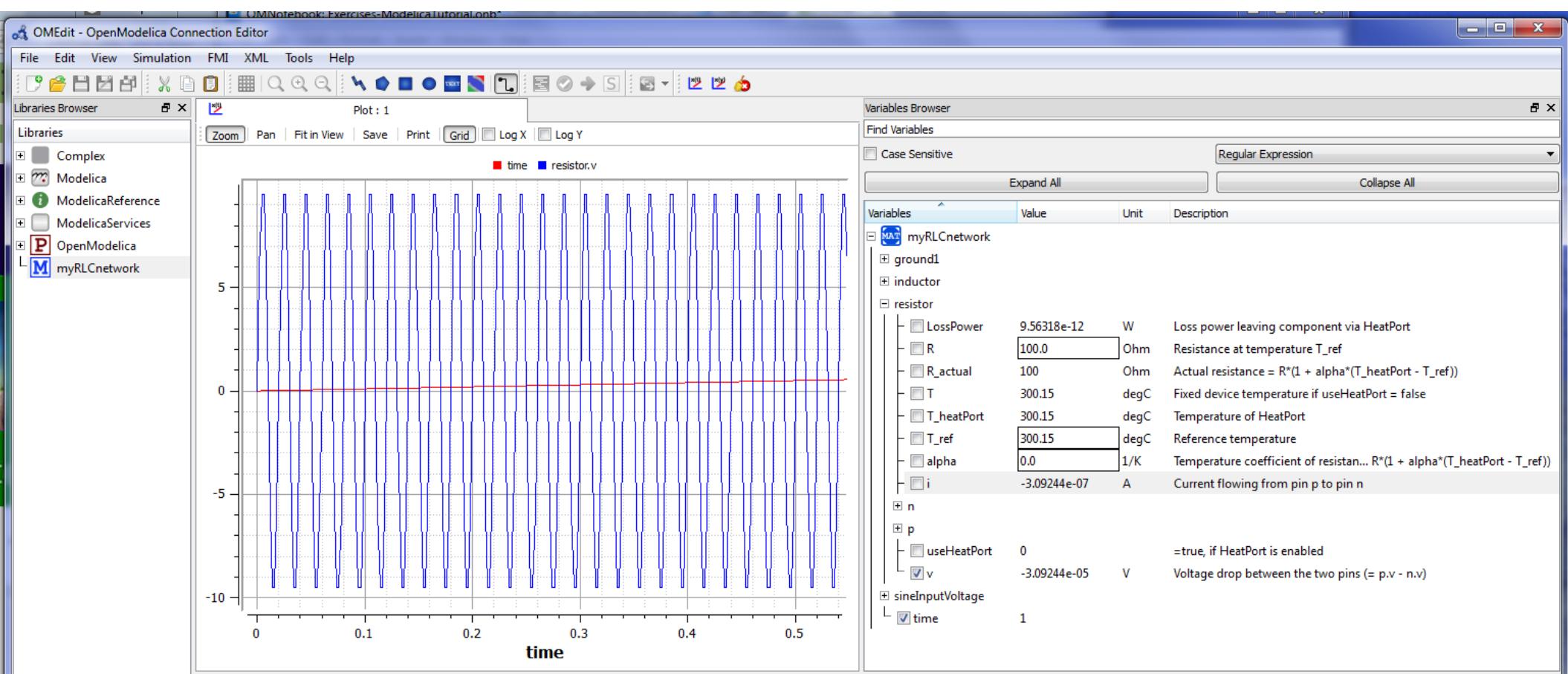
|   |               |                       |          |
|---|---------------|-----------------------|----------|
|  29/09/20...   | C File        | myRLCnetwork          | 14 KB    |
|  29/09/20...   | Application   | myRLCnetwork          | 9,960 KB |
|  29/09/20...   | LIBS File     | myRLCnetwork.libs     | 0 KB     |
|  29/09/20...   | Text Document | myRLCnetwork          | 0 KB     |
|  29/09/20...   | MAKEFILE File | myRLCnetwork.makefile | 2 KB     |
|  29/09/20...   | O File        | myRLCnetwork.o        | 17 KB    |
|  29/09/20...   | C File        | myRLCnetwork_01exo    | 2 KB     |
|  29/09/20...   | O File        | myRLCnetwork_01exo.o  | 2 KB     |
|  29/09/20...   | C File        | myRLCnetwork_02nls    | 2 KB     |
|  29/09/20...   | O File        | myRLCnetwork_02nls.o  | 1 KB     |
|  29/09/20...   | C File        | myRLCnetwork_03isy    | 2 KB     |
|  29/09/20...   | O File        | myRLCnetwork_03isy.o  | 1 KB     |
|  29/09/20...   | C File        | myRLCnetwork_04set    | 2 KB     |
|  29/09/20...   | O File        | myRLCnetwork_04set.o  | 1 KB     |
|  29/09/20...   | C File        | myRLCnetwork_05evt    | 3 KB     |
|  29/09/20...   | O File        | myRLCnetwork_05evt.o  | 2 KB     |
|  29/09/20...   | C File        | myRLCnetwork_06inz    | 7 KB     |
|  29/09/20...   | O File        | myRLCnetwork_06inz.o  | 5 KB     |
|  29/09/20...   | C File        | myRLCnetwork_07dly    | 2 KB     |
|  29/09/20...   | O File        | myRLCnetwork_07dly.o  | 1 KB     |
|  29/09/20...  | C File        | myRLCnetwork_08bnd    | 7 KB     |
|  29/09/20... | O File        | myRLCnetwork_08bnd.o  | 5 KB     |
|  29/09/20... | C File        | myRLCnetwork_09alg    | 2 KB     |
|  29/09/20... | O File        | myRLCnetwork_09alg.o  | 1 KB     |
|  29/09/20... | C File        | myRLCnetwork_10asr    | 2 KB     |
|  29/09/20... | O File        | myRLCnetwork_10asr.o  | 1 KB     |
|  29/09/20... | C File        | myRLCnetwork_11mix    | 2 KB     |
|  29/09/20... | H File        | myRLCnetwork_11mix.h  | 0 KB     |
|  29/09/20... | O File        | myRLCnetwork_11mix.o  | 1 KB     |
|  29/09/20... | C File        | myRLCnetwork_12jac    | 4 KB     |
|  29/09/20... | H File        | myRLCnetwork_12jac.h  | 2 KB     |



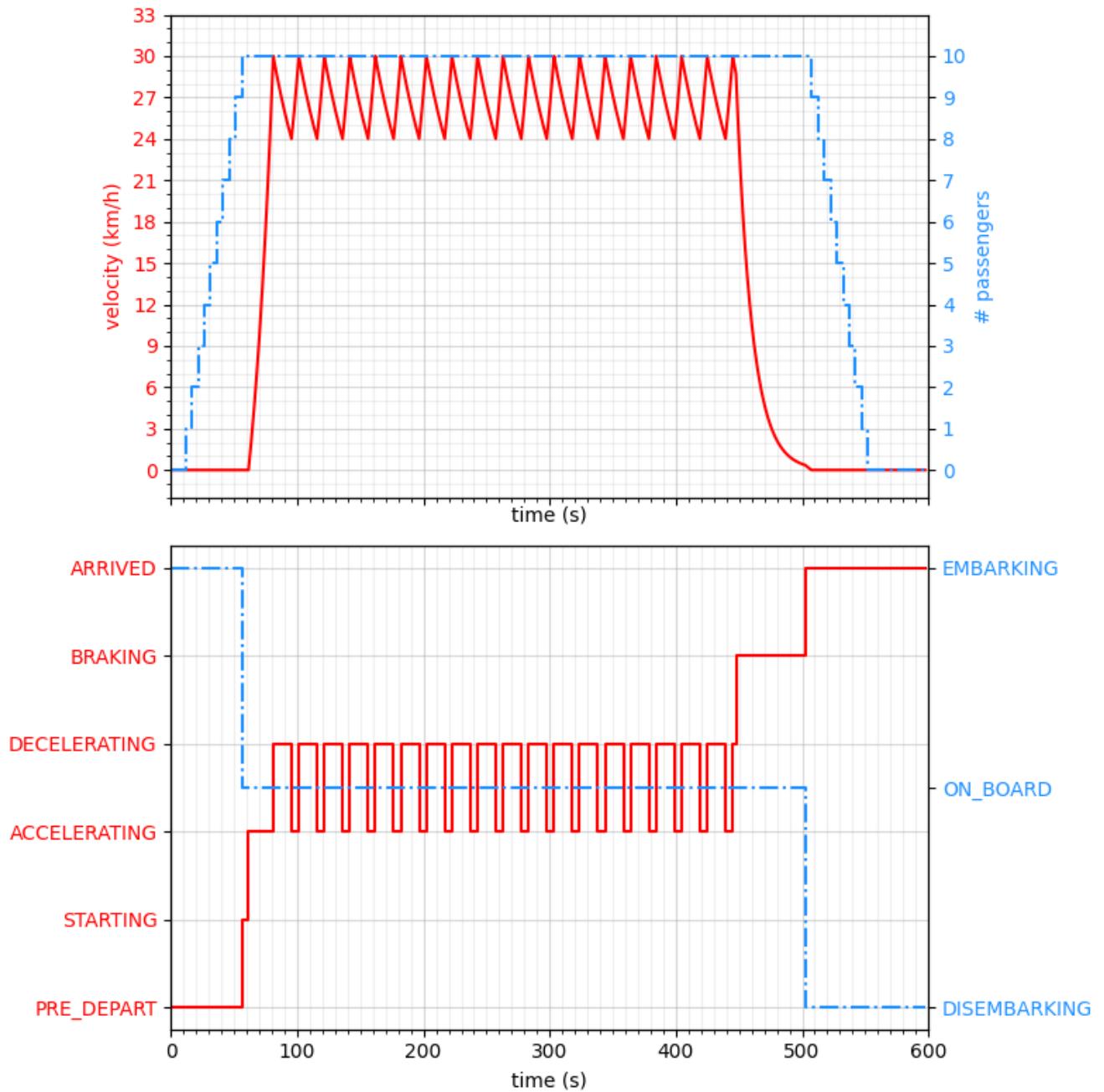
OMEdit - myRLCnetwork Simulation Output

Output    Compilation

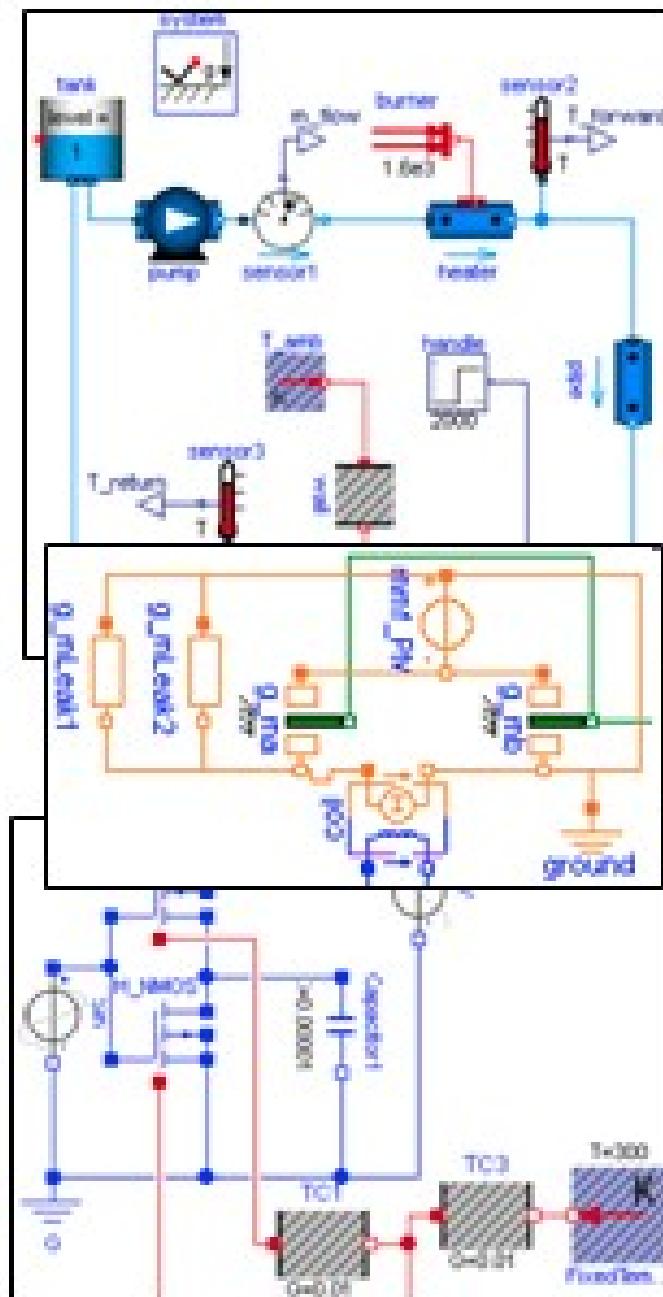
```
C:/Users/hv/AppData/Local/Temp/OpenModelica/OMEdit/myRLCnetwork.exe -port=49502 -logFormat=xml -w -lv=LOG_STATS
LOG_STATS | info    | ### STATISTICS ####
LOG_STATS | info    | timer
| | | 0.0150538s [ 46.9%] pre-initialization
| | | 4.18139e-005s [ 0.1%] initialization
| | | 2.0907e-005s [ 0.1%] steps
| | | 0.0157118s [ 49.0%] creating output-file
| | | 0.000115558s [ 0.4%] event-handling
| | | 0.000295738s [ 0.9%] overhead
| | | 0.000824114s [ 2.6%] simulation
| | | 0.0320637s [100.0%] total
LOG_STATS | info    | events
| | | 0 state events
| | | 0 time events
LOG_STATS | info    | solver: DASSL
| | | 2431 steps taken
| | | 3266 calls of functionODE
| | | 165 evaluations of jacobian
| | | 73 error test failures
| | | 0 convergence test failures
LOG_STATS | info    | ### END STATISTICS ###
```



# Hybrid



# MODELICA Standard Library (MSL)



# Controller Design and Tuning